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COUPLED BASE MOTION RESPONSE ANALYSIS OF PAYLOAD STRUCTURAL SYSTEMS

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"COUPLED BASE MOTION RESPONSE ANALYSIS
OF PAYLOAD STRUCTURAL SYSTEMS"

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CONTENTS

Foreward

This report, prepared by the Structural Dynamics Group, Department of Civil Engineering, College of Engineering and Applied Science, University of Colorado under contract NAS8-31619, presents the results of a study that investigated the conceptual approach termed "Coupled Base Motion Response Analysis of Payload Structural Systems". The study was performed from July 1975 through March 1976 and is presented in three sections.

Section 1. Analytical Developments

Section 2. Example problem typifying HEAO-MECO event

Section 3. Program Code

The study was administered by the National Aeronautics and Space Administration's George C. Marshall Space Flight Center, Huntsville, Alabama, under the technical direction of Mr. Wayne Holland.

Principal Nomenclature

s, S	Support system
B	Branch system
B'	Primed branch system
M_{1s}, M_{2s}	Masses of the support system
M_{1B}, M_{2B}	Masses of the branch system
$[M]_B$	Mass of the cantilevered branch system
K_s	Stiffness of support spring
K_B	Stiffness of branch spring
$K_{B'}$	Stiffness of primed branch spring
C_s	Viscous damping coefficient for support system
C_B	Viscous damping coefficient for branch system
$C_{B'}$	Viscous damping coefficient for primed branch system
$\rho_s, \lceil\rho\rfloor_S$	Damping ratio for support system
$\rho_B, \lceil\rho\rfloor_B$	Damping ratio for branch system
$\rho_{B'}$	Damping ratio for primed branch system
$\lceil\omega\rfloor_s, \lceil\omega\rfloor_S$	Circular frequency of support system
$\lceil\omega\rfloor_B$	Circular frequency of branch system
$\lceil\omega\rfloor_{B'}$	Circular frequency of primed branch system
$[\varphi_s], [\varphi']_s$	Reduced mode shapes of support system with respect to interface
$[\varphi_1^c]_B, [\varphi_C]_B$	Mode shapes of cantilevered branch system
$[\varphi_1^c]_{B'}$	Mode shapes of primed cantilevered branch system
$[\beta]$	Collapse transformation matrix
$[\Gamma]$	Loads transformation that relates interface forces to the branch point load vector

x, y, z	Coordinate directions
x_1, z_1, δ, etc.	Discrete deflections
θ_y	Discrete rotation
$\ddot{x}_1, \ddot{z}_1, \ddot{\theta}_y, \ddot{\delta}$, etc.	Discrete accelerations
q	Modal deflection
$\ddot{q}$	Modal accelerations
$Q_E(t), Q_{ES}(t)$	Applied forcing function
$F_I(t)$	Interface forcing function
$\ddot{\delta}_{FE}(t)$	Interface acceleration function
AE	Shear stiffness
EI	Bending stiffness

Abstract

This document details analytical procedures and discusses example problem results for coupled base motion response analysis of payload structural systems.

The report is divided into three major sections.

I Analytical Development

II Demonstration Example

III Program Code

A systems analysis program is described which by component analysis structural transient response analyses can be completed. Sophisticated aerospace structural systems require detail modeling and subsequent load and response analyses. The detail required consumes large amounts of computer time and a significant number of analysis samples. The subject analytical program presents a proven technique, used initially on the Skylab program, which is designed to reduce cost and schedule time on detail structural analyses of Structural Payload Systems.

Launch vehicles are composed of numerous structural components only one of which is normally defined as the payload structural system. However, to complete detail structural analyses, the entire structural model must be analyzed and the present custom is to reanalyze the entire system when the payload module or any component is revised, design model updated, or configuration updated. Detail or finite element modeling of large degree of freedom systems require significant amounts of computer time as well as complex models which are difficult at best for the engineer

or analyst to evaluate. Base motion procedures can be employed where critical segments of complex structural systems or components may be analyzed for various load conditions ithout having to re-establish the entire structural system coupled model properties.

The transient response characteristics of a complex structural system can be used as a basis for evaluating the transient response of a similar system, identical to the original except for changes in the dynamic representation of one or more of the original subsystems.

The study is mathematically derived in Appendix A and illustrated step by step by a demonstration problem followed by a more detailed model typifying the HEAO-MECO event.

I. Analytical Development

A. Discussion

Large complex structural systems create special problems for the structural dynamist. Launch systems, Atlas/Centaur, Titan, and Space Shuttle, have varied payload classes which can be accomodated as well as numerous mission events, conditions, and configurations. These payloads require detail dynamics analysis to predict their design loads and accelerations. However, due to the inherent complexity of the structural systems, dynamic response analyses of the integrated system for each payload will necessarily be expensive and time consuming.

The Skylab program had similar problems and requirements. Due in large part to the complexity of the docking latch and probe assembly a "base motion" method was derived to calculate orbital loads (Refs. 1, 2, and 3) using supplied docking interface forces which were implied to the latch connection points. Numerous analyses were completed improving the technique of applying interface or "base" forces to a structure culminating in Skylab Base Motion Analysis Report (Ref. 4), which defined loads analyses for Skylab launch configuration events.

This culminated into the prime base motion technical approach (Refs. 5 and 6) by which incompatible base motion excitations are used. This led to the development, presented herein, of an analytical computer program for coupled base motion analysis of payload structural systems. The methodology described in Appendix A was used to develop the associated computer program. Liberal

use of reference 6 was used to formulate this methodology.

B. Technical Approach

The primary function of the derived analytical program is to take the method described in detail in Appendix A, evaluate its application, make modifications where required, and define a methodology whereby payload structural systems may be analyzed using a coupled base motion approach.

This was accomplished by breaking the study into three distinct steps or cases for analysis. The subject cases compose three primary branch systems identified in Figure 1.

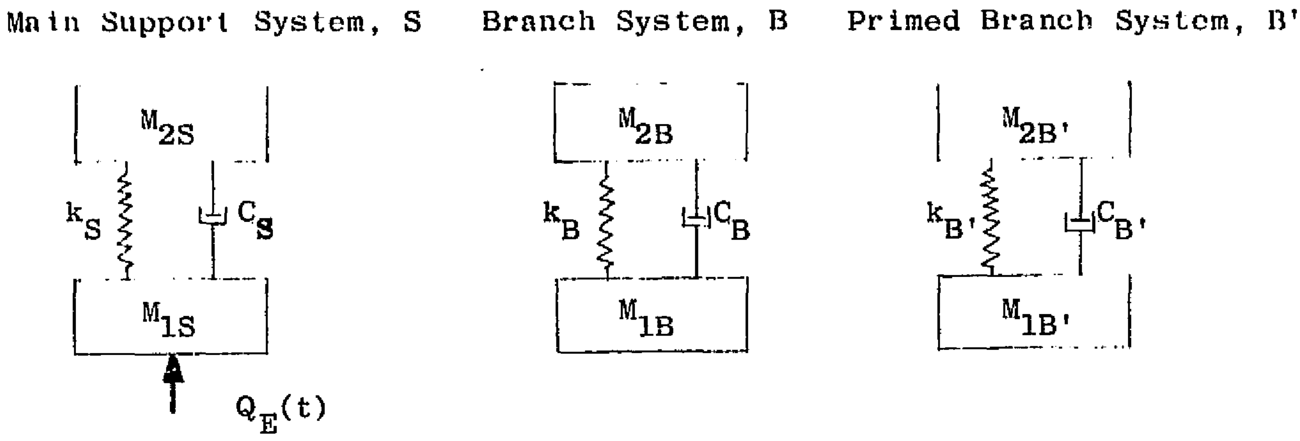


Figure 1 Primary Base Motion Systems

Utilizing these primary systems and applying the methodology contained within Appendix A a system of equations and techniques are derived which define an analytical method for coupled base motion response analyses of payload structural systems.

Two cases are described to define a combination of payload effects and which make the basis for the derived methodology (see Figure 2).

Case A Evaluate response of the main support systems due to applied forcing function $Q_E(t)$. We now include structural effects of the branch system B and evaluate the total response from the known response of the support system, s.

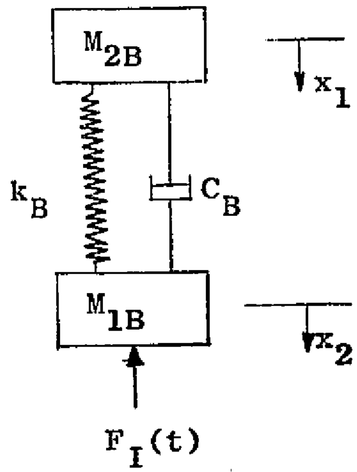
Case B Evaluate the response of the primed branch system, B', by attaching it to the main support system, s. This is accomplished by removing the dynamic contribution of the branch system, B, from the total response.

Case A is seen as the exact solution. Therefore Case B results correlated with the exact solution will yield the relative merit of the method.

The basic analytical approach of this study is to take the sample cases, Figure 2, apply these cases to a "hand crank" data evaluation using simplified data, Figure 3, to evaluate the analytical technique. The generation of the equations of motion for the subject study has been completed and is presented herein. A more sufficient demonstration model is included in Section II.

CASE A

Branch B representing the payload is defined by



(prescribed base excitation is $\ddot{x}_2$)

(Interface Force)

The response equations are:

$$\left[\begin{array}{c|c} M_{1B} & 0 \\ \hline 0 & M_{2B} \end{array}\right] \left\{\begin{array}{c} \ddot{x}_2 \\ \hline \ddot{x}_1 \end{array}\right\} + \left[\begin{array}{c|c} C_B & -C_B \\ \hline -C_B & C_B \end{array}\right] \left\{\begin{array}{c} \dot{x}_2 \\ \hline \dot{x}_1 \end{array}\right\}$$

$$+ \left[\begin{array}{c|c} k_B & -k_B \\ \hline -k_B & k_B \end{array}\right] \left\{\begin{array}{c} x_2 \\ \hline x_1 \end{array}\right\} = \left\{\begin{array}{c} -F_I(t) \\ \hline 0 \end{array}\right\} \tag{1}$$

From the method suggested in reference 5, the equation of motion for this system subjected to base motion is:

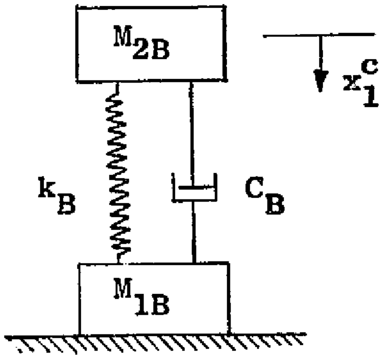
$$\left[M_{2B}\right] \left\{\ddot{x}_1^c\right\} + \left[C_B\right] \left\{\dot{x}_1^c\right\} + \left[k_B\right] \left\{x_1^c\right\} = - \left[M_{2B}\right] \left[\beta\right] \left\{\ddot{x}_2(t)\right\} \tag{2}$$

and

$$\left[K_{22}\right] = \left[k_B\right] , \left[K_{21}\right] = \left[-k_B\right]$$

$$\left[\beta\right] = - \left[K_{22}\right]^{-1} \left[K_{21}\right] = \left[1\right]$$

$$\therefore \quad \left\{x_1\right\} = \left[\beta\right] \left\{x_2\right\} + \left\{x_1^c\right\}$$



Taking $\quad \{x_1^c\}_B = [\varphi_1^c]_B \{q\}_B$

where $\quad [\varphi_1^c]_B = [1/\sqrt{M_{2B}}] = 1$

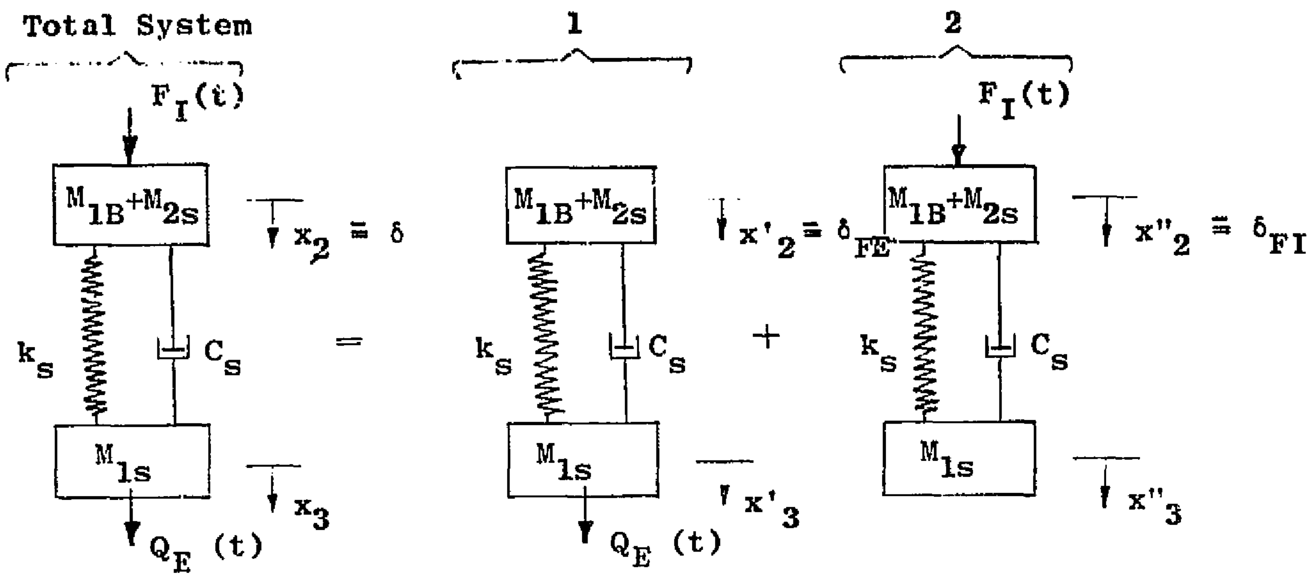
and $\quad [\ulcorner \omega^2 \lrcorner]_B = [k_B/M_{2B}] = 9000.$ (3)

or $\quad [\ulcorner \omega \lrcorner]_B = 94.8683$

then

$$\{x_1\}_B = [\bar{1}] \{x_2\} + [1] \{q\}_B$$
$$\{x_1\}_B = [\beta] \{\delta(t)\} + [\varphi_1^c] \{q\}_B \tag{4}$$

M_{1B} and M_{2s} are subjected to inertia forces at the interface and to account for the total inertia force the two masses are lumped together as a part of the support system, booster.



The response equation for 1 of the system is:

$$\begin{bmatrix} M_{1B}+M_{2s} & 0 \\ 0 & M_{1s} \end{bmatrix} \begin{Bmatrix} \ddot{x}''_2 \\ \ddot{x}''_3 \end{Bmatrix} + \begin{bmatrix} C_s & -C_s \\ -C_s & C_s \end{bmatrix} \begin{Bmatrix} \dot{x}''_2 \\ x''_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \begin{Bmatrix} F_I(t) \\ 0 \end{Bmatrix} \tag{5}$$

where

$$M_{1B} + M_{2s} = 0.5 + 1.5 = 2 \text{ slug/in}$$

$$M_{1s} = 2 \text{ slug/in}$$

$$k_s = 1000 \text{ lb/in}$$

For an undamped system without external excitation

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}''_2 \\ \ddot{x}''_3 \end{Bmatrix} + 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \dot{x}''_2 \\ \dot{x}''_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

and

$$\begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \left[\varphi\right]_s \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix}$$

From modal analysis we get:

$$\left[\varphi\right]_s = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix}$$

$$\left[\diagdown \omega^2 \diagdown\right]_s = \begin{bmatrix} 0 & 0 \\ 0 & 10000 \end{bmatrix} \quad \text{and} \quad \left[\diagdown \omega \diagdown\right]_s = \begin{bmatrix} 0 & 0 \\ 0 & 100 \end{bmatrix} \tag{6}$$

Thus Eq. (5) becomes

$$\left\{\ddot{q}\right\}_s + \left\lceil 2\rho\omega \right\rfloor_s \left\{\dot{q}\right\}_s + \left\lceil \omega^2 \right\rfloor_s \left\{q\right\}_s = \left[\varphi_s\right]^T \left\{F_I(t)\right\} \tag{7}$$

where

$$\left\{\ddot{q}\right\} = \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix}$$

and

$$\left[\varphi_s\right]^T = \left[\varphi\right]_s^T \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.5 \\ 0.5 \end{Bmatrix} \tag{8}$$

We also know

$$\left\{\ddot{\delta}(t)\right\} = \left\{\ddot{\delta}_{FE}(t)\right\} + \left\{\ddot{\delta}_{FI}(t)\right\} \quad \text{and} \quad \left\{\ddot{x}_2(t)\right\} = \left\{\ddot{x}'_2(t)\right\} + \left\{\ddot{x}''_2(t)\right\}$$

or

$$\left\{\ddot{\delta}_{FI}(t)\right\} = \left\{\ddot{x}''_2\right\} = \left[\varphi_1\right]_s \left\{\ddot{q}\right\}_s$$

where

$$\left[\varphi_1\right]_s = \left[0.5 \quad 0.5\right] \tag{9}$$

$$\left\{\ddot{x}_2(t)\right\} = \left\{\ddot{x}'_2(t)\right\} + \left[\varphi_1\right]_s \left\{\ddot{q}\right\}_s = \left\{\ddot{\delta}_{FE}(t)\right\} + \left[\varphi_1\right]_s \left\{\ddot{q}\right\}_s \tag{10}$$

The interface forcing function $\{F_I(t)\}$ is determined by solving the booster dynamic equation.

Interface Force $\{F_I(t)\}$

For the branch, the response equations with interface force acting are

$$
\begin{aligned}
M_{1B}\ddot{x}_2 - C_B\left(\dot{x}_1 - \dot{x}_2\right) - k_B\left(x_1 - x_2\right) &= - F_I(t) \\
M_{2B}\ddot{x}_1 + C_B\left(\dot{x}_1 - \dot{x}_2\right) + k_B\left(x_1 - x_2\right) &= 0
\end{aligned}
\tag{11}
$$

For inertially relieved system, $\ddot{x}_2 = 0$, from (11) we obtain

$$C_B\left(\dot{x}_1 - \dot{x}_2\right) + k_B\left(x_1 - x_2\right) = F_I(t)$$

Taking $\quad \left[\Gamma\right]\left\{C_B\left(\dot{x}_1 - \dot{x}_2\right) + k_B\left(x_1 - x_2\right)\right\} = F_I(t)$

$$\left[\Gamma\right] = \left[1\right]$$

and from Eq. (11) we get

$$\left\{F_I(t)\right\} = - \left[\Gamma\right]\left[M_{2B}\right]\left\{\ddot{x}_1\right\}$$

Substituting for Eq. (4) obtain

$$= - \left[\Gamma\right]\left[M_{2B}\right]\left(\left[\beta\right]\left\{\ddot{\delta}(t)\right\} + \left[\varphi_1^c\right]_B\left\{\ddot{q}\right\}_B\right)$$

Substituting for Eq. (10) obtain

$$= - \left[\Gamma\right]\left[M_{2B}\right]\left(\left[\beta\right]\left\{\ddot{\delta}_{FE}(t)\right\} + \left[\beta\right]\left[\varphi_1\right]_S\left\{\ddot{q}\right\}_S + \left[\varphi_1^c\right]_B\left\{\ddot{q}\right\}_B\right)$$

and

$$\left\{F_I(t)\right\} = \left[TB\right]\left\{\ddot{q}\right\}_B + \left[TS\right]\left\{\ddot{q}\right\}_S + \left[TI\right]\left\{\ddot{\delta}_{FE}\right\} \tag{12}$$

where

$$\underset{(1x1)}{\left[TB\right]} = -\left[\Gamma\right]\left[M_{2B}\right]\left[\varphi_1^c\right]_B = -\left[1\right]\left[1\right]\left[1\right] \tag{13a}$$

$$= \left[-1\right]$$

$$\underset{(1x2)}{\left[TS\right]} = -\left[\Gamma\right]\left[M_{2B}\right]\left[\beta\right]\left[\varphi_1\right]_S = -\left[1\right]\left[1\right]\left[1\right]\left[0.5 \quad 0.5\right] \tag{13b}$$

$$= -\left[0.5 \quad 0.5\right]$$

$$\left[TI\right] = -\left[\Gamma\right]\left[M_{2B}\right]\left[\beta\right] = -\left[1\right]\left[1\right]\left[1\right] \tag{13c}$$

$$= \left[-1\right]$$

For payload or branch, from (2) using modal coordinates:

$$\left\{\ddot{q}\right\}_B + \left[\diagdown 2\rho\omega \diagdown\right]_B \left\{\dot{q}\right\}_B + \left[\diagdown \omega^2 \diagdown\right]\left\{q\right\}_B =$$

$$-\left[\varphi_1^c\right]_B^T\left[M_{2B}\right]\left[\beta\right]\left\{\ddot{x}_2(t)\right\}$$

Substituting for Eq. (10) obtain

$$\begin{aligned}\left\{\ddot{q}\right\}_B = &-\left[\varphi_1^c\right]_B^T\left[M_{2B}\right]\left[\beta\right]\left\{\ddot{\delta}_{FE}(t)\right\} \\ &-\left[\varphi_1^c\right]_B^T\left[M_{2B}\right]\left[\beta\right]\left[\varphi_1\right]_S\left\{\ddot{q}\right\}_S\end{aligned} \tag{14}$$

and for the support or booster:

$$\left\{\ddot{q}\right\}_S + \left\lceil 2\rho\omega \right\rfloor_S \left\{\dot{q}\right\}_S + \left\lceil \omega^2 \right\rfloor_S \left\{q\right\}_S = \left[\varphi_S\right]^T \left\{F_I(t)\right\}$$

Substituting for Eq. (12) obtain

$$\begin{aligned} \left\{\ddot{q}\right\}_S = {} & \left[\varphi_S\right]^T \left[TB\right] \left\{\ddot{q}\right\}_B + \left[\varphi_S\right]^T \left[TS\right] \left\{\ddot{q}\right\}_S \\ & + \left[\varphi_S\right]^T \left[TI\right] \left\{\ddot{\delta}_{FE}(t)\right\} \end{aligned} \tag{15}$$

Eqs. (14) and (15) can be written in matrix form as

$$\begin{aligned} & \left[\begin{array}{c|c} \left\lceil I \right\rfloor & \left[\varphi_1^c\right] \left[M_{2B}\right] \left[\beta\right] \left[\varphi_1\right]_S \\ \hline -\left[\varphi_S\right]^T \left[TB\right] & \left\lceil I \right\rfloor - \left[\varphi_S\right]^T \left[TS\right] \end{array}\right] \left\{\begin{array}{c} \ddot{q}_B \\ \hline \ddot{q}_S \end{array}\right\} + \left[\begin{array}{c|c} \left\lceil 2\rho\omega \right\rfloor_B & 0 \\ \hline 0 & \left\lceil 2\rho\omega \right\rfloor_S \end{array}\right] \left\{\begin{array}{c} \dot{q}_B \\ \hline \dot{q}_S \end{array}\right\} \\ & + \left[\begin{array}{c|c} \left\lceil \omega^2 \right\rfloor_B & 0 \\ \hline 0 & \left\lceil \omega^2 \right\rfloor_S \end{array}\right] \left\{\begin{array}{c} q_B \\ \hline q_S \end{array}\right\} = \left[\begin{array}{c} -\left[\varphi_1^c\right]_B^T \left[M_{2B}\right] \left[\beta\right] \\ \hline \left[\varphi_S\right]^T \left[TI\right] \end{array}\right] \left\{\ddot{\delta}_{FE}(t)\right\} \end{aligned} \tag{16}$$

The various matrices are calculated below:

$$\left[\varphi_1^c\right]_B^T \left[M_{2B}\right] \left[\beta\right] \left[\varphi_1\right]_S = \left[1\right] \left[1\right] \left[1\right] \left[0.5 \quad 0.5\right]$$

$$= \left[0.5 \quad 0.5\right]$$

$$-\left[\varphi_S\right]^T \left[TB\right] = -\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \left[-1\right] = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$\left[I \right] - \left[\varphi_s\right]^T \left[TS\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1.25 \end{bmatrix}$$

$$- \left[\varphi_1^c\right]_B^T \left[M_{2B}\right] \left[\beta\right] = - \left[1\right] \left[1\right] \left[1\right] = \left[-1\right]$$

$$\left[\varphi_s\right]^T \left[TI\right] = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \left[-1\right] = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

Substituting these into Eq. (16) finally gives

$$\begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1.25 & 0.25 \\ 0.5 & 0.25 & 1.25 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} 1.8973 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} 9000 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 10000 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{bmatrix} -1 \\ -0.5 \\ -0.5 \end{bmatrix} \left\{\ddot{\delta}_{FE}(t)\right\}$$

The solution to this equation requires simply the definition of $\{\delta_{FE}(t)\}$ which can be defined from the system described under 1 of page 12.

To find $\left\{\ddot{\delta}_{FE}(t)\right\}$ or $\left\{\ddot{x}'_2(t)\right\}$ the response equation for support is

$$\begin{bmatrix} M_{2s}+M_{1B} & 0 \\ 0 & M_{1s} \end{bmatrix} \begin{Bmatrix} \ddot{x}'_2 \\ \ddot{x}'_3 \end{Bmatrix} + \begin{bmatrix} C_s & -C_s \\ -C_s & C_s \end{bmatrix} \begin{Bmatrix} \dot{x}'_2 \\ \dot{x}'_3 \end{Bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ Q_E\ (t) \end{Bmatrix}$$

Taking modal coordinates

$$\begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix} = \left[\varphi\right]_s \begin{Bmatrix} \xi_2 \\ \xi_3 \end{Bmatrix}$$

The previous equation reduces to the form; using Eqs. (6)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\xi}_2 \\ \ddot{\xi}_3 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \dot{\xi}_2 \\ \dot{\xi}_3 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 10000 \end{bmatrix} \begin{Bmatrix} \xi_2 \\ \xi_3 \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} Q_E\ (t)$$

or in simple form

$$\left\{\ddot{\xi}\right\} + \left\lceil 2\rho\omega \right\rfloor_s \left\{\dot{\xi}\right\} + \left\lceil \omega^2 \right\rfloor_s \left\{\xi\right\} = \left[\varphi'_s\right]^T \left\{Q_E\ (t)\right\} \tag{17}$$

where $\left[\varphi'_s\right]^T = \left[\varphi\right]_s^T \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

$$\left\{\ddot{\delta}_{FE}(t)\right\} = \left\{\ddot{x}'_2\right\} = \left[\varphi_1\right]_S \begin{Bmatrix} \ddot{\xi}_2 \\ \ddot{\xi}_3 \end{Bmatrix} = \left[0.5 \quad 0.5\right] \begin{Bmatrix} \ddot{\xi}_2 \\ \ddot{\xi}_3 \end{Bmatrix} \tag{18}$$

Thus $\ddot{\delta}_{FE}(t)$ can be calculated solving Eqs. (17) and (18). The final deflection of the system can then be defined by

$$\begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix} + \begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \left[\varphi\right]_S \begin{Bmatrix} \xi_2 \\ \xi_3 \end{Bmatrix} + \left[\varphi\right]_S \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix} \tag{19}$$

where $\begin{Bmatrix} \xi_2 \\ \xi_3 \end{Bmatrix}$ are obtained from solving Eq. (17) and $\begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix}$ are obtained from solving Eq. (16).

$$\text{Finally} \quad \left\{x_1\right\} = \left[\beta\right] \left\{x_2\right\} + \left[\varphi_1^c\right]_B \left\{q\right\}_B$$

$$= \left[1\right] \left\{x_2\right\} + \left[1\right] \left\{q_1\right\} \tag{20}$$

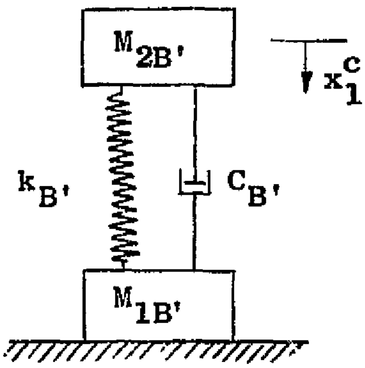
$$= \left\{x_2\right\} + \left\{q_1\right\}$$

where $\left\{q_1\right\}$ is found from Eq. (16) and $\left\{x_2\right\}$ from Eq. (19)

CASE B

Branch B' representing the new or revised payload. The base motion excitation is represented similar to Case A.

$$\left[M_{2B'}\right]\left\{\ddot{x}_1^c\right\} + \left[C'_B\right]\left\{\dot{x}_1^c\right\} + \left[k'_B\right]\left\{x_1^c\right\} = -\left[M_{2B'}\right]\left[\beta\right]\left\{\ddot{x}_2\right\} \tag{21}$$



and as before

$$\left\{x_1\right\} = \left[\beta\right]\left\{x_2\right\} + \left\{x_1^c\right\}$$

Taking modal coordinates for Eq. (21)

$$\left\{x_1^c\right\} = \left[\varphi_1^c\right]_{B'}\left\{q\right\}_{B'}$$

Eq. (21) reduces to

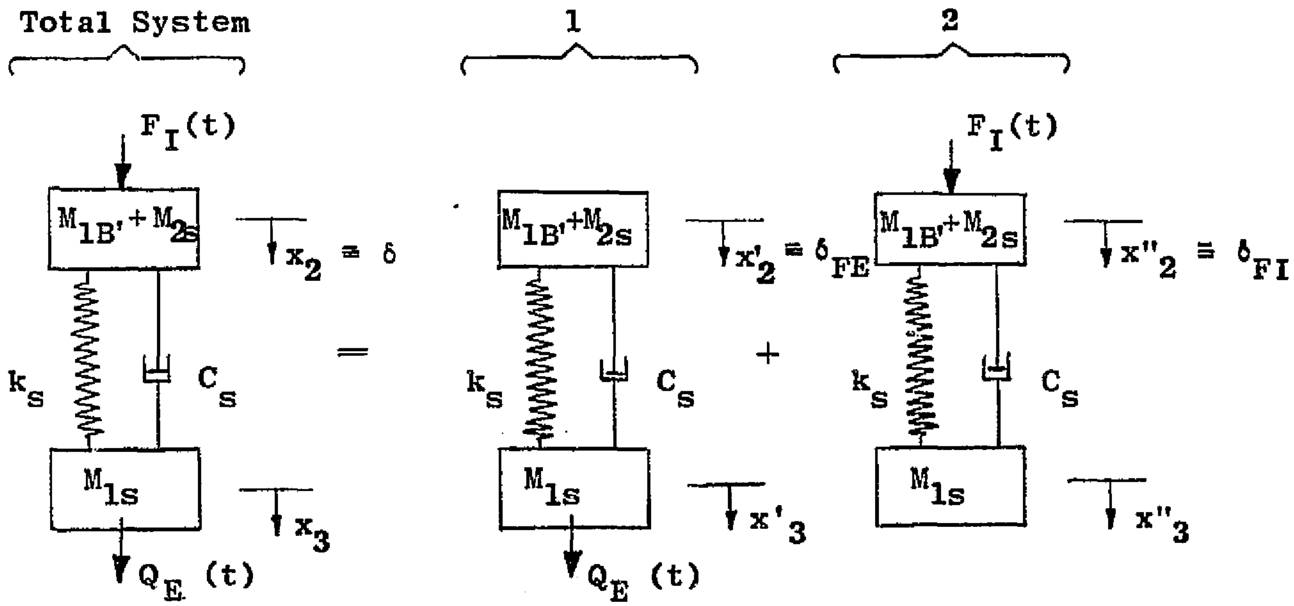
$$\left\{\ddot{q}\right\}_{B'} + \left[\diagdown 2\rho\omega \diagdown\right]_{B'}\left\{\dot{q}\right\}_{B'} + \left[\diagdown \omega^2 \diagdown\right]_{B'}\left\{q\right\}_{B'}$$
$$= -\left[\varphi_1^c\right]_{B'}^T\left[M_{2B'}\right]\left[\beta\right]\left\{\ddot{x}_2\right\} \tag{22}$$

$$\left[\varphi_1^c\right]_{B'} = \left[\frac{1}{\sqrt{M_{2B'}}}\right] = \left[\frac{1}{\sqrt{1.5}}\right] = \left[.8165\right]$$

$$\left[\omega^2\right]_{B'} = \left[\frac{k_{B'}}{M_{2B'}}\right] = \left[\frac{9000}{1.5}\right] = \left[6000\right] \tag{23}$$

$$\left[2\rho\omega\right]_{B'} = \left[\sqrt{k_{B'}/M_{2B'}}\right] = \left[1.5492\right]$$

and lumping the interface masses in the booster to define the support system we have as before:



The response equation for Case 2 is:

$$\begin{bmatrix} M_{1B'}+M_{2s} & 0 \\ 0 & M_{1s} \end{bmatrix} \begin{Bmatrix} \ddot{x}''_2 \\ \ddot{x}''_3 \end{Bmatrix} + \begin{bmatrix} C_s & -C_s \\ -C_s & C_s \end{bmatrix} \begin{Bmatrix} \dot{x}''_2 \\ \dot{x}''_3 \end{Bmatrix}$$
$$+ \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \begin{Bmatrix} F_I(t) \\ 0 \end{Bmatrix} \tag{24}$$

$M_{1B'} + M_{2s} = 2.0$ slug/in.

$M_{1s} = 2$ slug/in.

$k_s = 10^4$ lb/in.

For undamped system without external excitation

$$\begin{bmatrix} 2.0 & 0 \\ 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{x}''_2 \\ \ddot{x}''_3 \end{Bmatrix} + 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Taking

$$\begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \left[\varphi\right]_s \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix}$$

From modal analysis:

$$\left[\varphi\right]_s = \begin{bmatrix} 0.50 & 0.50 \\ 0.50 & -0.50 \end{bmatrix}$$

$$\left[\ \omega^2 \searrow\right]_s = \begin{bmatrix} 0 & 0 \\ 0 & 10000 \end{bmatrix} \qquad \left[\ 2\rho\omega \searrow\right]_s = \begin{bmatrix} 0 & 0 \\ 0 & 2.0 \end{bmatrix} \tag{25}$$

Thus Eq. (24) becomes.

$$\{\ddot{q}\}_s + \left[\ 2\rho\omega \searrow\right]_s \{\dot{q}\}_s + \left[\ \omega^2 \searrow\right]_s \{q\}_s = \left[\varphi_s\right]^T \{F_I(t)\} \tag{26}$$

Where $\left[\varphi_s\right]^T = \left[\varphi\right]_s^T \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Also $\{\ddot{\delta}_{FI}(t)\} = \ddot{x}''_2 = \left[\varphi_1\right]_s \{\ddot{q}\}_s$

where $\left[\varphi_1\right]_s = \left[0.5 \quad 0.5\right]$ (27)

We also know

$$\{\ddot{\delta}(t)\} = \{\ddot{\delta}_{FE}(t)\} + \{\delta_{FI}(t)\} \quad \text{and} \quad \{\ddot{x}_2(t)\} = \{\ddot{x}'_2(t)\} + \{\ddot{x}''_2(t)\}$$
$$= \{\ddot{\delta}_{FE}(t)\} + \left[\varphi_1\right]_s \{\ddot{q}\}_s \tag{28}$$

Interface Force $F_I(t)$

As in Case A, we will have

$$\left[\Gamma\right] = \left[1\right]$$

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$$\left\{F_I(t)\right\} = \left[TB\right]\left\{\ddot{q}\right\}_{B'} + \left[TS\right]\left\{\ddot{q}\right\}_S + \left[TI\right]\left\{\ddot{\delta}_{FE}\right\} \tag{29}$$

Since $M_{2B'} = 1.5$ Slug/in. we will have

$$\left[TB\right] = -\left[\Gamma\right]\left[M_{2B'}\right]\left[\varphi_1^c\right]_{B'} = -\left[1\right]\left[1.5\right]\left[.8165\right] = \left[-1.22475\right] \tag{30a}$$

$$\left[TS\right] = -\left[\Gamma\right]\left[M_{2B'}\right]\left[\beta\right]\left[\varphi_1\right]_S = -\left[1\right]\left[1.5\right]\left[1\right]\left[0.5 \quad 0.5\right] \tag{30b}$$

$$= -\left[0.75 \quad 0.75\right]$$

$$\left[TI\right] = -\left[\Gamma\right]\left[M_{2B'}\right]\left[\beta\right] = -\left[1\right]\left[1.5\right]\left[1\right] = \left[-1.5\right] \tag{30c}$$

Eq. (16) will be similar for this case with subscript B changed to B'

$$\left[\varphi_1^c\right]_{B'}^T \left[M_{2B'}\right]\left[\beta\right]\left[\varphi_1\right]_S = \left[.8165\right]\left[1.5\right]\left[1\right]\left[0.5 \quad 0.5\right]$$

$$= \left[0.6124 \quad 0.6124\right]$$

$$-\left[\varphi_S\right]^T\left[TB\right] = -\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\left[-1.22475\right] = \begin{bmatrix} 0.6124 \\ 0.6124 \end{bmatrix}$$

$$\left[\diagdown I \diagdown\right] - \left[\varphi_S\right]^T\left[TS\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\left[0.75 \quad 0.75\right]$$

$$= \begin{bmatrix} 1.375 & 0.375 \\ 0.375 & 1.375 \end{bmatrix}$$

$$- \left[\varphi_1^c\right]_{B'}^T \left[M_{2B'}\right] \left[\beta\right] = - \left[.8165\right] \left[1.5\right] \left[1\right] = \left[-1.22475\right]$$

$$\left[\varphi_s\right]^T \left[TI\right] = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \left[-1.5\right] = \begin{bmatrix} 0.75 \\ 0.75 \end{bmatrix}$$

Substituting these in Eq. (16) we obtain our final equation as

$$\begin{bmatrix} 1.0 & 0.6124 & 0.6124 \\ 0.6124 & 1.375 & 0.375 \\ 0.6124 & 0.375 & 1.375 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} 1.5492 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} 6000.0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 10{,}000 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{bmatrix} -1.22475 \\ -0.75 \\ -0.75 \end{bmatrix} \left\{\ddot{\delta}_{FE}(t)\right\} \tag{31}$$

As previously noted in the response analysis of Case A (page 18) the solution to this equation requires simply the definition of $\{\ddot{\delta}_{FE}(t)\}$.

<u>To Find $\left\{\ddot{\delta}_{FE}(t)\right\}$</u>

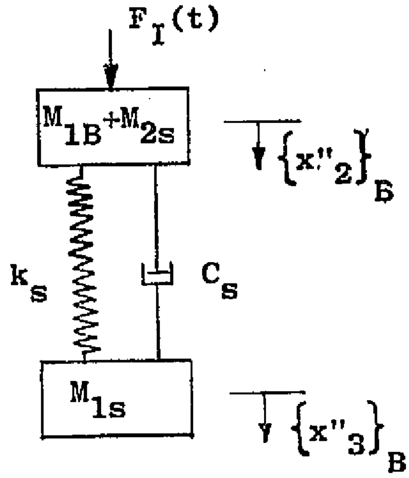
From Case A we know:

$\left\{F_I(t)\right\}$

and $\left\{\ddot{\delta}(t)\right\} = \left\{\ddot{x}_2(t)\right\}$

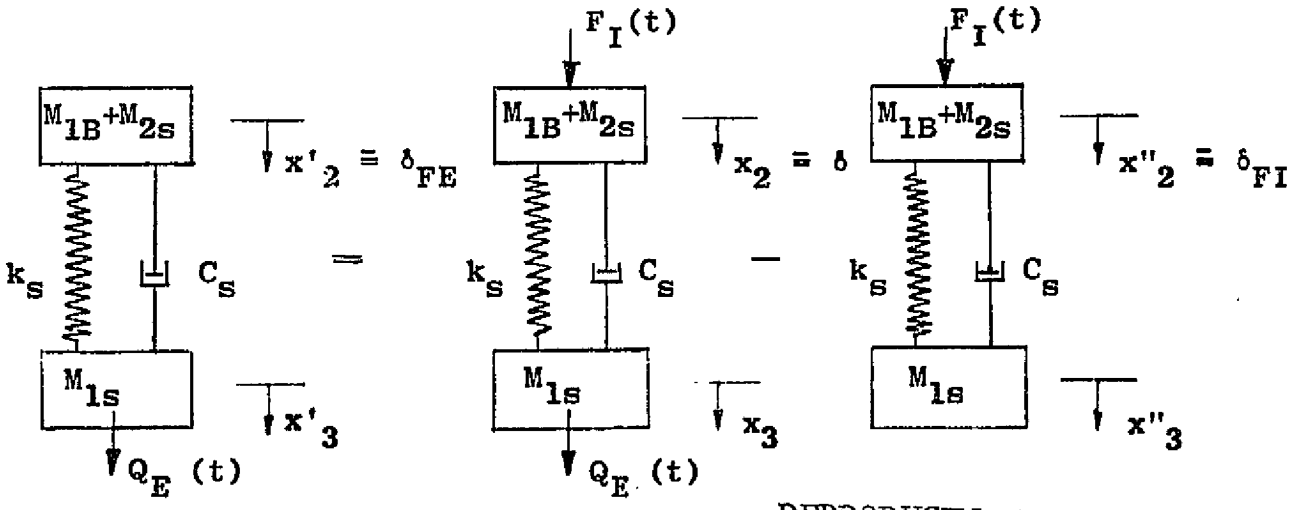
To find $\{\ddot{\delta}_{FE}\}$ the following steps are performed:

(1) Analyze support of Case A with $F_I(t)$ only acting



$$\begin{bmatrix}2&0\\0&2\end{bmatrix}\begin{Bmatrix}\ddot{x}''_2\\\ddot{x}''_3\end{Bmatrix}+\begin{bmatrix}C_s&-C_s\\-C_s&C_s\end{bmatrix}\begin{Bmatrix}\dot{x}''_2\\\dot{x}''_3\end{Bmatrix}+10^4\begin{bmatrix}1&-1\\-1&1\end{bmatrix}\begin{Bmatrix}x''_2\\x''_3\end{Bmatrix}=\begin{Bmatrix}1\\0\end{Bmatrix}\{F_I(t)\} \qquad (32)$$

(2) Subtract the results of Eq. (32) from the total response of support of Case A



$$\left\{\ddot{\delta}_{FE}(t)\right\} = \left\{\ddot{x}'_2\right\} = \left\{\ddot{x}_2\right\} - \left\{\ddot{x}''_2\right\} = \left\{\ddot{x}_2\right\} - \left[\varphi_1\right]_s \left\{\ddot{q}\right\}_s \tag{33}$$

The final deflections for the system can then be written as:

$$\begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix} + \begin{Bmatrix} x''_2 \\ x''_3 \end{Bmatrix} = \begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix} + \left[\varphi\right]_s \begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix} \tag{34}$$

where $\begin{Bmatrix} x'_2 \\ x'_3 \end{Bmatrix}$ = Deflections obtained from step 2 for finding $\left\{\delta_{FE}\right\}$

$\begin{Bmatrix} q_2 \\ q_3 \end{Bmatrix}$ = Deflections obtained from solution of Eq. (31)

and

$$\begin{aligned} \left\{x_1\right\} &= \left[\beta\right] \left\{x_2\right\} + \left[\varphi_1^c\right]_{B'} \left\{q\right\}_{B'} \\ &= \left[1\right] \left\{x_2\right\} + \left[0.8165\right] \left\{q_1\right\} \\ &= \left\{x_2\right\} + 0.8165 \left\{q_1\right\} \end{aligned} \tag{35}$$

where $\left\{q_1\right\}$ is found from Eq. (31) and $\left\{x_2\right\}$ from Eq. (34).

C. Conclusions

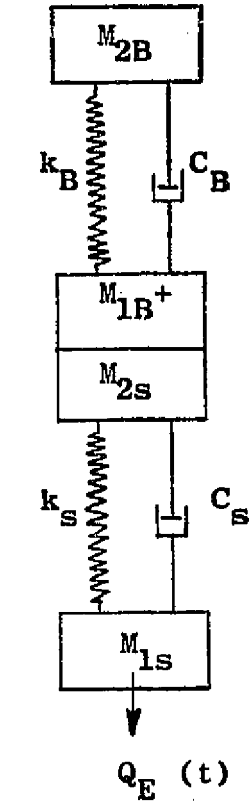
During the course of evaluating the results, it was found that the details associated with modelling the interface required

further consideration. In particular, it is noted that application of the previously described approach converged to the exact solution when the mass on the branch side of the interface was made an integral part of the support system, s. That is to say, the branch side of the interface contains a stiffness element between the interface and the first branch mass element. This fact does not impose serious restrictions on the basic approach, This effect is abridged by utilization of a massless interface for the branch side.

The results obtained for Cases A and B are concisely summarized in Tables 1 and 2, and Figures 1 through 5. As indicated in the Figures for Case A, the curves are virtually the same. The same was also true for Case B, hence these figures are not included. Correlation of the two cases indicate errors less than 3%. Actually some individual errors were higher but seriously affected by roundoff error.

At this stage of development the method has been sufficiently validated to proceed with a more detailed analytical model which is defined in Section II.

CASE A



CASE B

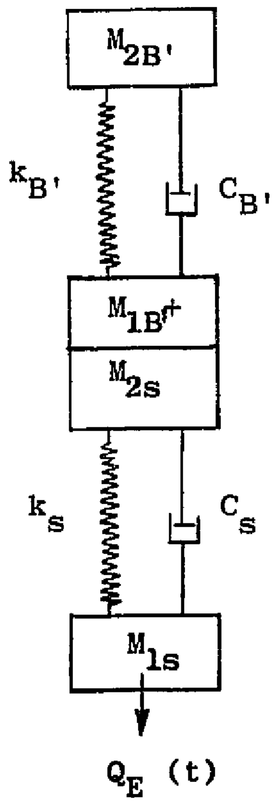
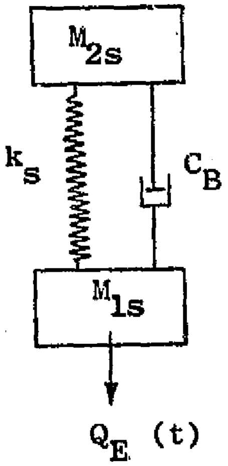
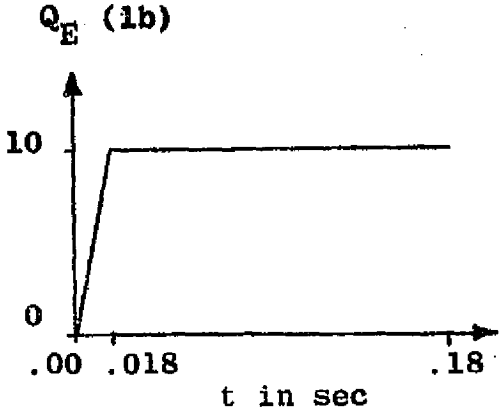


FIGURE 2 Study Program Sample Cases

SUPPORT S



Variation of Q_E (t):



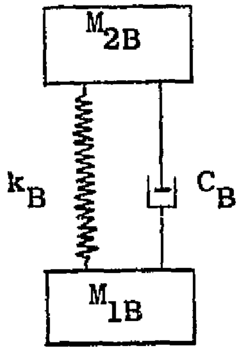
M_{2s} = 1.5 slug/in

M_{1s} = 2.0 slug/in

k_s = 1×10^4 lb/in

ρ_s = 0.01

BRANCH B



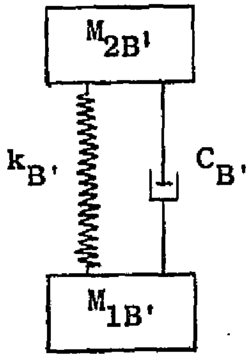
M_{2B} = 1.0 slug/in

M_{1B} = 0.5 slug/in

k_B = 0.9×10^4 lb/in

ρ_B = 0.01

BRANCH B'



$M_{2B'}$ = 1.5 slug/in

$M_{1B'}$ = 0.5 slug/in

$k_{B'}$ = 0.9×10^4 lb/in

$\rho_{B'}$ = 0.01

FIGURE 3 Study Data

FIGURE 4

Case A - $\ddot{x}_1$ Vs Time

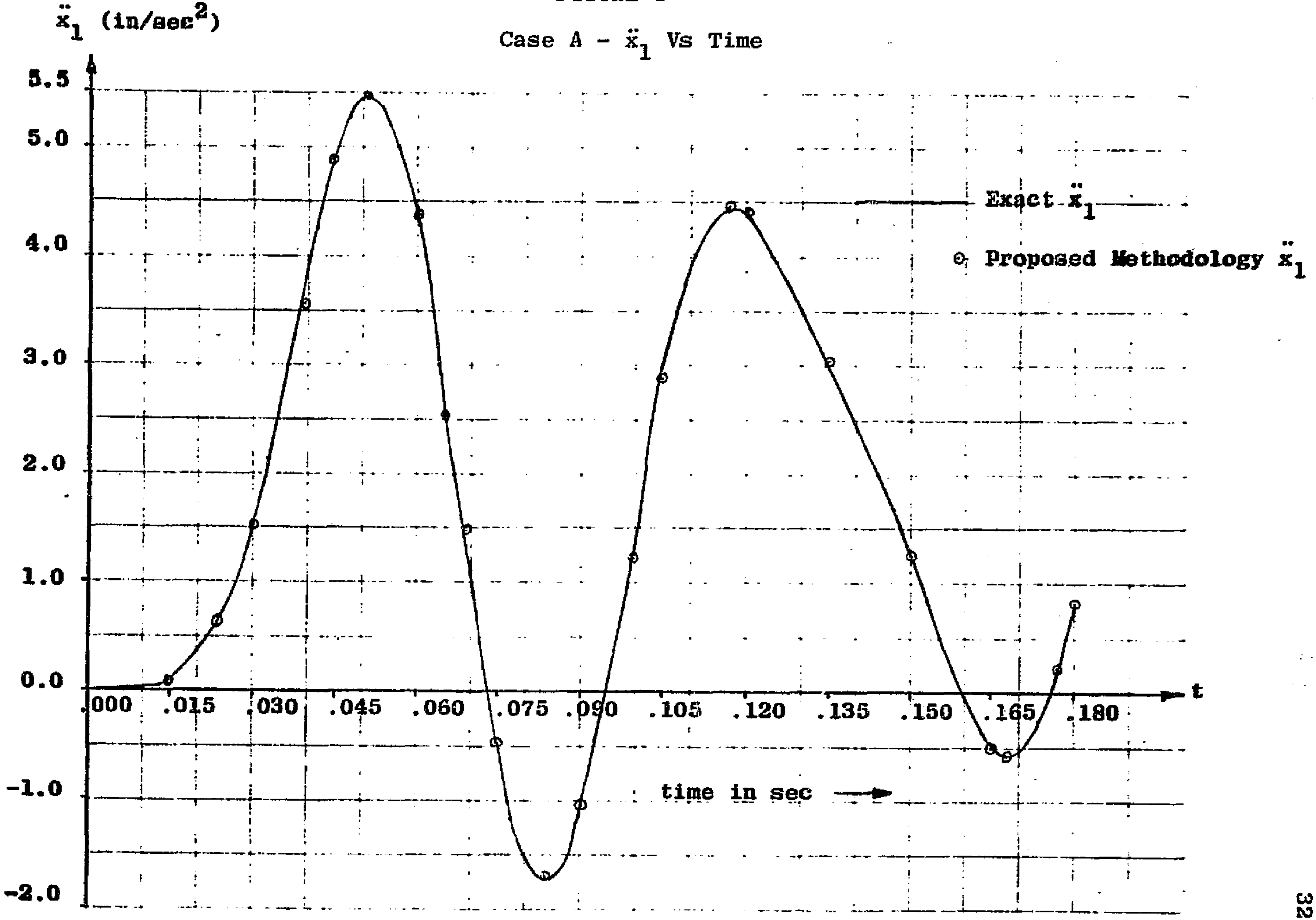


FIGURE 5

Case A - $\dot{x}_1$ Vs Time

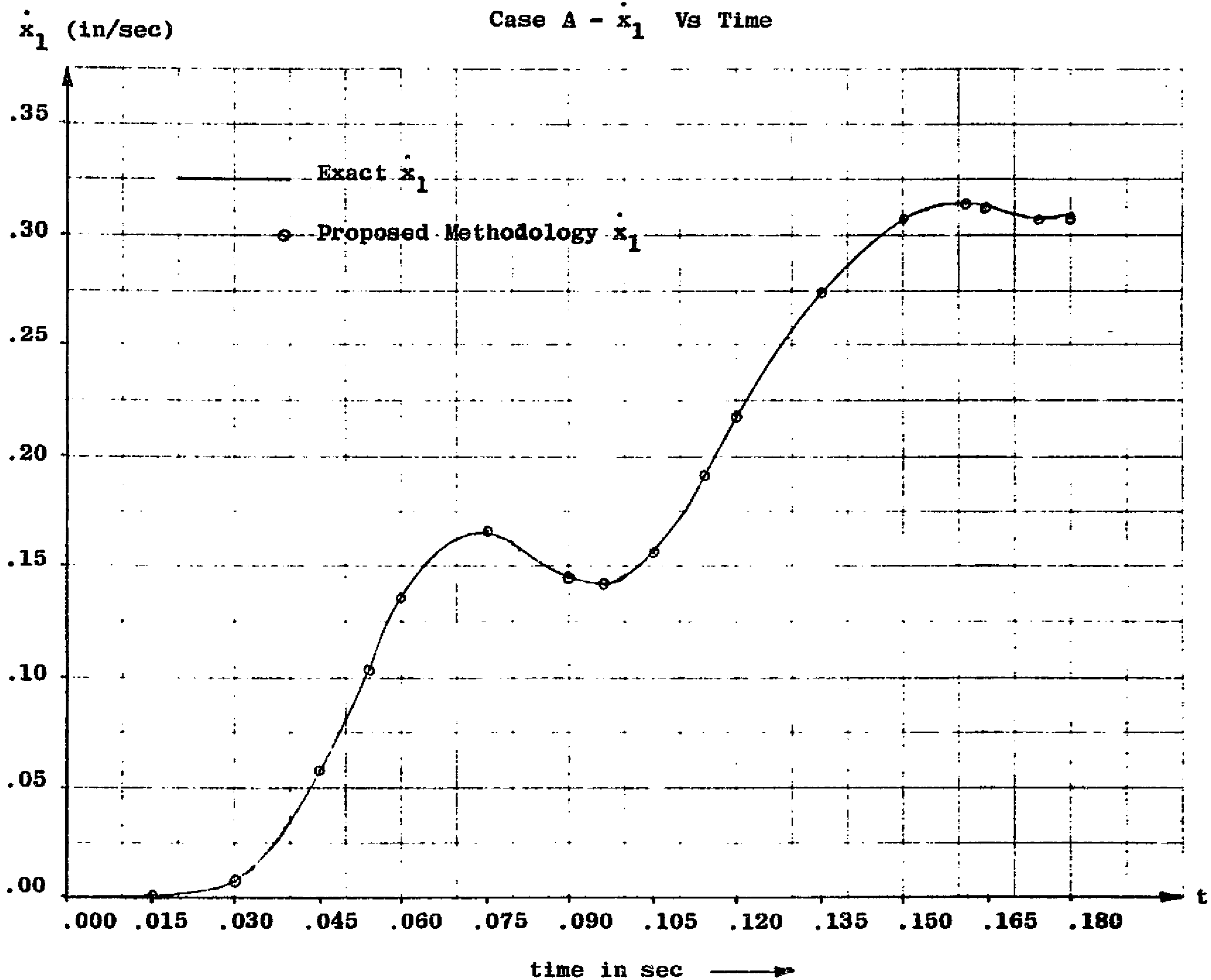


FIGURE 6

Case A - x_1 Vs Time

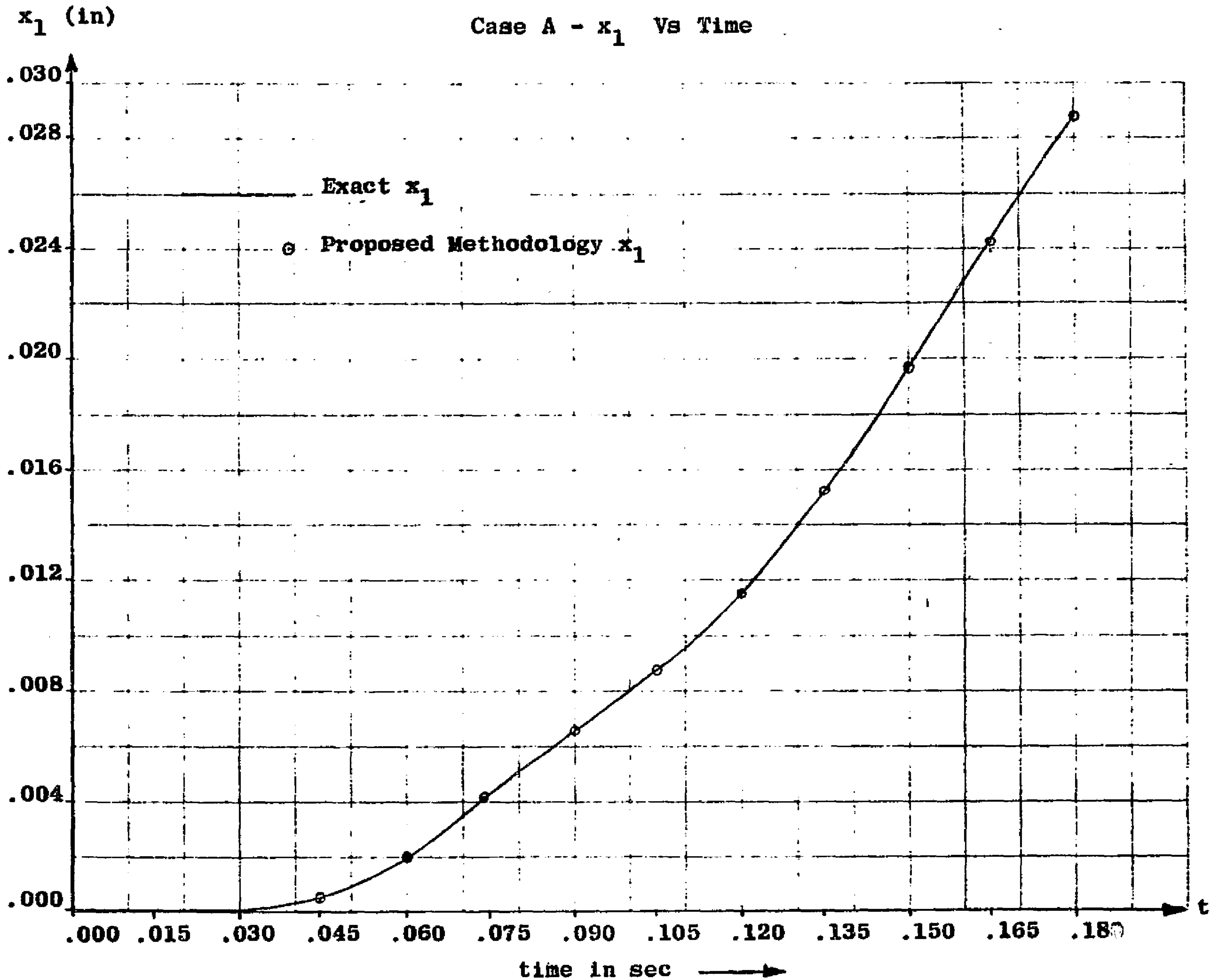


FIGURE 7

Case A - x_3 Vs Time

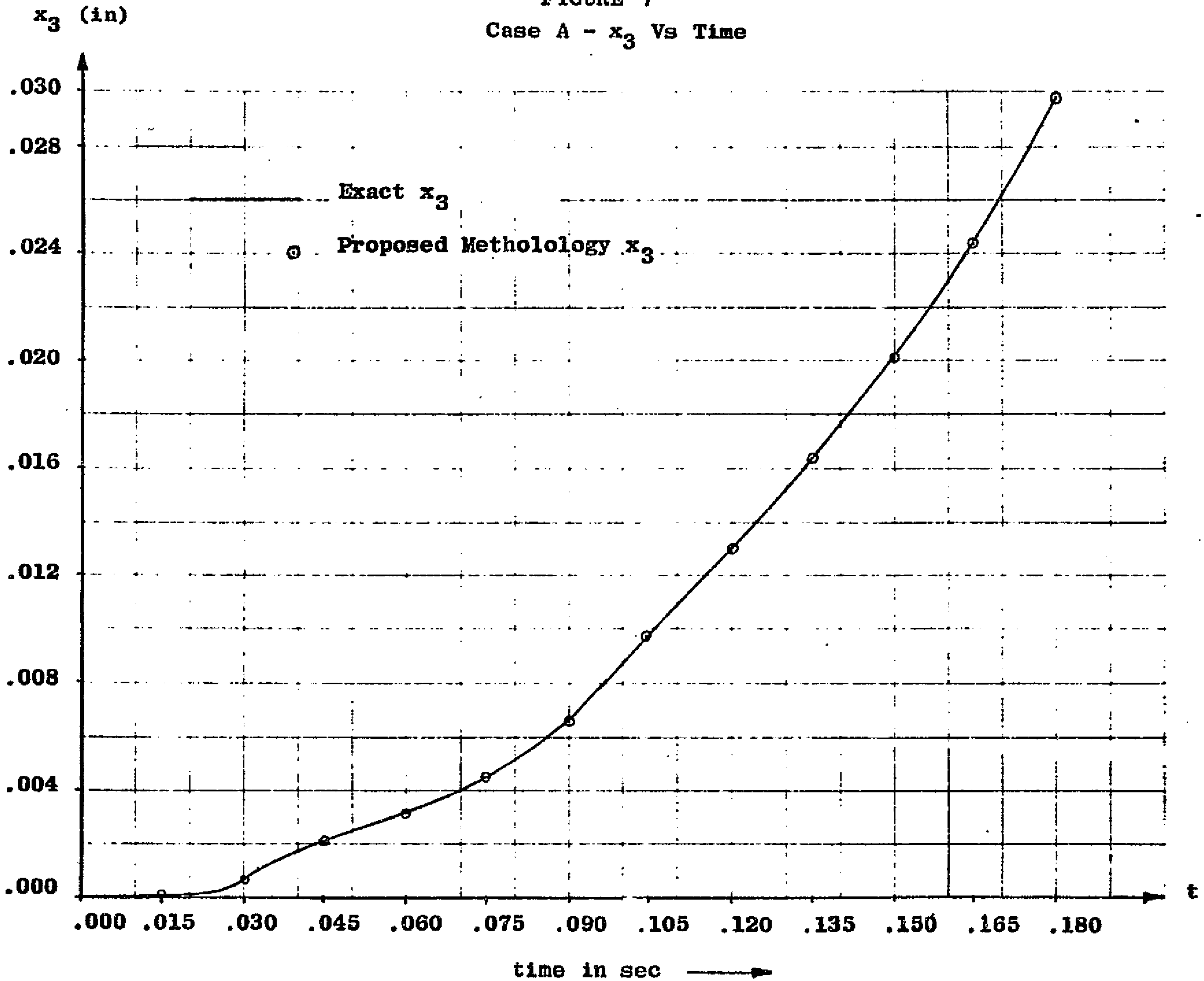


FIGURE 8

Case A - x_2 Vs. Time

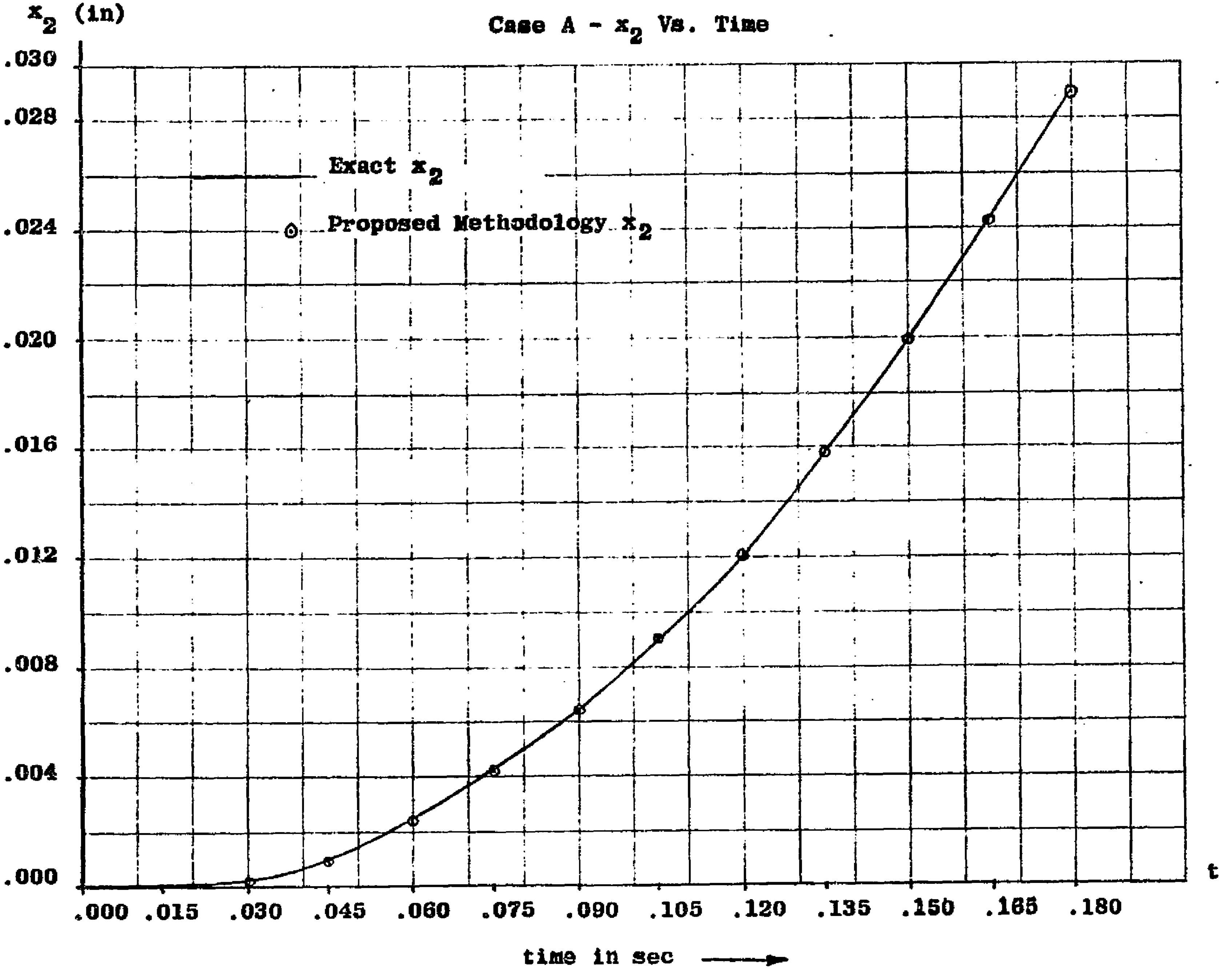


TABLE I

Case A (Exact *)

Time	A_1	A_2	A_3	V_1	V_2	V_3	D_1	D_2	D_3
.018	.1855 *.1875	1.083 1.085	3.824 3.821	4.846-4 6.469-4	3.352-3 5.354-3	3.934-2 3.932-2	8.104-7 1.923-6	2.040-5 2.038-5	2.488-4 2.487-4
.030	1.546 *1.532	2.915 2.924	1.312 1.310	9.081-3 9.188-3	2.986-2 2.997-2	7 050 -2 7.044-2	4.181-5 4.494-5	2.092-4 2.095-4	9.386-4 9.380-4
.045	4.851 *4.850	2.790 2.777	-2.153-1 -2.019-1	5.753-2 5.724-2	7.674-2 7.684-2	7.453-2 7.454-2	4.770-4 4.786-4	1.012-3 1.014-3	2.055-3 2.054-3
.060	4.373 *4.398	1.599 1.577	1.214 1.224	1.349-1 1.348-1	1.080-1 1.077-1	7.961-2 7.985-2	1.929-3 1.927-3	2.421-3 2.422-3	3.184-3 3.184-3
.075	-4.395-1** *-4.605-1	2.028 2.067	3.191 3.163	1.647-1 1.649-1	1.342-1 1.340-1	1.134-1 1.135-1	4.270-3 4.269-3	4.228-3 4.225-3	4.594-3 4.598-3
.090	-1.105 *-1.108	1.685 1.702	3.867 3.851	1.441-1 1.440-1	1.641-1 1.645-1	1.688-1 1.685-1	6.599-3 6.598-3	6.472-3 6.471-3	6.698-3 6.699-3
.105	2.884 *2.937	1.269 1.213	2.289 2.319	1.566-1 1.568-1	1.840-1 1.840-1	2.178-1 2.176-1	8.776-3 8.776-3	9.091-3 9.096-3	9.627-3 9.624-3
.120	4.426 *4.387	2.850 2.867	-7.217-2 -6.015-2	2.169-1 2.173-1	2.137-1 2.132-1	2.329-1 2.332-1	1.155-2 1.156-2	1.204-2 1.204-2	1.305-2 1.305-2
.135	3.073 *3.001	3.013 3.062	4.507-1 4.375-1	2.744-1 2.738-1	2.619-1 2.620-1	2.309-1 2.310-1	1.526-2 1.527-2	1.561-2 1.560-2	1.652-2 1.652-2
.150	1.254 *1.324	1.046 1.005	3.327 3.333	3.072-1 3.067-1	2.922-1 2.924-1	2.591-1 2.592-1	1.966-2 1.965-2	1.980-2 1.980-2	2.014-2 2.014-2
.165	-5.032-1 *-4.466-1	1.199 1.186	4.053 4.037	3.115-1 3.121-1	3.052-1 3.048-1	3.190-1 3.191-1	2.433-2 2.433-2	2.428-2 2.428-2	2.446-2 2.447-2
.180	8.704-1 *8.117-1	2.559 2.619	2.005 1.975	3.087-1 3.091-1	3.346-1 3.348-1	3.661-1 3.657-1	2.896-2 2.896-2	2.905-2 2.905-2	2.964-2 2.964-2
Time Max	.051 *5.460	.129 3.414	.159 4.206	.159 3.132-1	.180 3.348-1	.180 3.348-1	.180 2.896-2	.180 2.905-2	.180 2.964-2
Time Max	.151 5.443	.129 3.360	.159 4.206	.159 3.130	.180 3.346	.180 3.661-1	.180 2.896-2	.180 2.905-2	.180 2.966-2

** 4.395-1 = 4.395×10^{-1}

N.B. Units: -

Time: sec

Acceleration A_1, A_2, A_3: in/sec^2

Velocity V_1, V_2, V_3: in/sec

Displacement D_1, D_2, D_3: in

TABLE II

Case B

Time (sec)	x_1 (in)			x_2 (in)			x_3 (in)		
	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|
.015	4.789×10^{-7}	-1.523×10^{-7}		8.532×10^{-6}	8.601×10^{-6}		1.474×10^{-4}	1.474×10^{-4}	
.030	3.176×10^{-5}	2.902×10^{-5}	9.44%	2.084×10^{-4}	2.086×10^{-4}	0.1 %	9.377×10^{-4}	9.386×10^{-4}	0.1 %
.045	3.506×10^{-4}	3.499×10^{-4}	0.20%	9.937×10^{-4}	9.923×10^{-4}	0.14%	2.051×10^{-3}	2.053×10^{-3}	0.1 %
.060	1.507×10^{-3}	1.501×10^{-3}	0.40%	2.289×10^{-3}	2.287×10^{-3}	0.09%	3.151×10^{-5}	3.152×10^{-3}	1.03%
.075	3.622×10^{-3}	3.622×10^{-3}	0.0 %	3.826×10^{-3}	3.829×10^{-3}	0.08%	4.415×10^{-3}	4.412×10^{-3}	0.07%
.090	6.020×10^{-3}	6.018×10^{-3}	0.03%	5.808×10^{-3}	5.811×10^{-3}	0.05%	6.147×10^{-3}	6.142×10^{-3}	0.08%
.105	8.223×10^{-3}	8.227×10^{-3}	0.05%	8.331×10^{-3}	8.326×10^{-3}	0.06%	8.610×10^{-3}	8.611×10^{-3}	0.01%
.120	1.059×10^{-2}	1.059×10^{-2}	0.0 %	1.110×10^{-2}	1.110×10^{-2}	0.0 %	1.183×10^{-2}	1.183×10^{-2}	0.0 %
.135	1.361×10^{-2}	1.361×10^{-2}	0.0 %	1.420×10^{-2}	1.421×10^{-2}	0.07%	1.535×10^{-2}	1.535×10^{-2}	0.0 %
.150	1.742×10^{-2}	1.741×10^{-2}	0.06%	1.793×10^{-2}	1.793×10^{-2}	0.0 %	1.878×10^{-2}	1.878×10^{-2}	0.0 %
.165	2.188×10^{-2}	2.188×10^{-2}	0.0 %	2.209×10^{-2}	2.208×10^{-2}	0.05%	2.241×10^{-2}	2.241×10^{-2}	0.0 %
.180	2.660×10^{-2}	2.261×10^{-2}	0.04%	2.646×10^{-2}	2.646×10^{-2}	0.0 %	2.676×10^{-2}	2.675×10^{-2}	0.04%
.180	2.660×10^{-2}*	2.261×10^{-2}*	0.04%						
.180				2.646×10^{-2}*	2.646×10^{-2}*	0.0 %	2.676×10^{-2}*	2.675×10^{-2}*	0.04%
.180									
	Average Error		0.92%	Average Error		0.06%	Average Error		0.04%

* Maximum Values

System Average = 0.34%

TABLE II (continued)

Case B

Time (sec)	$\dot{x}_1$ (in/sec)			$\dot{x}_2$ (in/sec)			$\dot{x}_3$ (in/sec)		
	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|
.015	1.816×10^{-4}	6.626×10^{-5}		2.725×10^{-3}	2.737×10^{-3}		2.839×10^{-2}	2.841×10^{-2}	
.030	6.524×10^{-3}	6.453×10^{-3}	1.1 %	2.971×10^{-2}	2.968×10^{-2}	0.1 %	7.040×10^{-2}	7.050×10^{-2}	0.14%
.045	4.317×10^{-2}	4.348×10^{-2}	0.71%	7.364×10^{-2}	7.350×10^{-2}	0.19%	7.398×10^{-2}	7.406×10^{-2}	0.11%
.060	1.131×10^{-1}	1.131×10^{-1}	0.0 %	9.486×10^{-2}	9.503×10^{-2}	0.18%	7.534×10^{-2}	7.510×10^{-2}	0.32%
.075	1.599×10^{-1}	1.596×10^{-1}	0.19%	1.134×10^{-1}	1.138×10^{-1}	0.44%	9.667×10^{-2}	9.636×10^{-2}	0.32%
.090	1.536×10^{-1}	1.539×10^{-1}	0.19%	1.525×10^{-1}	1.521×10^{-1}	0.26%	1.373×10^{-1}	1.375×10^{-1}	0.15%
.105	1.453×10^{-1}	1.456×10^{-1}	0.21%	1.792×10^{-1}	1.788×10^{-1}	0.22%	1.918×10^{-1}	1.921×10^{-1}	0.16%
.120	1.762×10^{-1}	1.756×10^{-1}	0.34%	1.915×10^{-1}	1.921×10^{-1}	0.31%	2.313×10^{-1}	2.312×10^{-1}	0.04%
.135	2.281×10^{-1}	2.276×10^{-1}	0.22%	2.264×10^{-1}	2.266×10^{-1}	0.09%	2.326×10^{-1}	2.326×10^{-1}	0.04%
.150	2.782×10^{-1}	2.789×10^{-1}	0.25%	2.674×10^{-1}	2.666×10^{-1}	0.3 %	2.289×10^{-1}	2.291×10^{-1}	0.08%
.165	3.119×10^{-1}	3.124×10^{-1}	1.16%	2.838×10^{-1}	2.839×10^{-1}	0.04%	2.623×10^{-1}	2.617×10^{-1}	0.23%
.180	3.123×10^{-1}	3.121×10^{-1}	0.06%	3.032×10^{-1}	3.041×10^{-1}	0.3 %	3.176×10^{-1}	3.170×10^{-1}	0.19%
.171	3.161×10^{-1}*	3.162×10^{-1}*	0.03%						
.180				3.032×10^{-1}*	3.041×10^{-1}*	0.3 %			
.180							3.176×10^{-1}*	3.170×10^{-1}*	0.19%
	Average Error		0.29%			0.22%			0.16%

* Maximum Values

System Average = 0.22%

TABLE II (concluded)

Case B

Time (sec)	$\ddot{x}_1$ (in/sec^2)			$\ddot{x}_2$ (in/sec^2)			$\ddot{x}_3$ (in/sec^2)		
	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|	Exact Soln.	Proposed Soln.	|% Error|
.015	6.02×10^{-2}	5.665×10^{-2}		6.778×10^{-1}	6.773×10^{-1}		3.444	3.447	
.030	1.103	1.113	0.9 %	2.869	2.856	2.0 %	1.303	1.309	0.46%
.045	3.897	3.901	0.1 %	2.371	2.380	0.4 %	-2.940×10^{-1}	-3.063×10^{-1}	4.0 %
.060	4.661	4.636	0.54%	7.886×10^{-1}	8.297×10^{-1}	4.95%	7.159×10^{-1}	6.931×10^{-1}	3.3 %
.075	1.157	1.173	1.36%	2.051	2.019	1.58%	2.082	2.102	0.95%
.090	-1.277	-1.244	2.65%	2.632	2.573	2.3 %	3.325	3.360	1.04%
.105	6.958×10^{-1}	6.413×10^{-1}	8.5 %	8.941×10^{-1}	9.564×10^{-1}	6.51%	3.584	3.563	0.59%
.120	3.123	3.082	1.33%	1.322	1.372	3.64%	1.336	1.316	1.52%
.135	3.534	3.616	2.27%	3.074	2.984	3.01%	-7.242×10^{-1}	-6.963×10^{-1}	4.0 %
.150	3.027	3.070	1.4 %	1.928	1.919	0.47%	8.021×10^{-1}	7.788×10^{-1}	3.0 %
.165	1.196	1.126	6.2 %	6.896×10^{-1}	7.919×10^{-1}	12.92%	3.413	3.364	1.46%
.180	-8.551×10^{-1}	-8.746×10^{-1}	2.23%	2.143	2.091	2.5 %	3.499	3.565	1.85%
.054	4.730*	4.920*	3.86%						
.036				3.117*	3.129*	0.4 %			
.018							3.824*	3.820*	0.1 %
	Average Error		2.61%			3.39%			1.86%

* Maximum Values

System Average = 2.62%

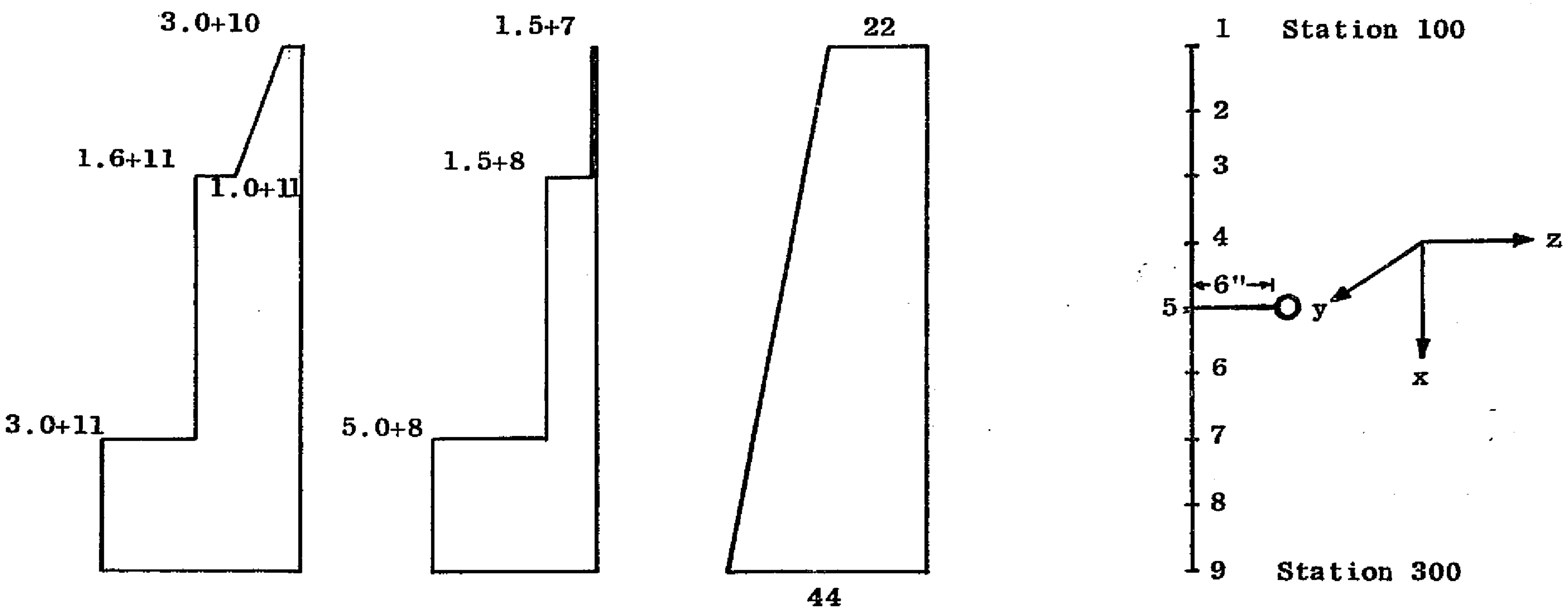
II. Demonstration Model

A. Introductory Remarks

A dynamic model is formulated to simulate a free-free vehicle in steady state accelerations. The transients of this model are examined due to application of external i) axial excitation and ii) axial and moment excitations. First, the complete modal system (i.e. integrated system) is analyzed as a basis for checking the results. Second, coupled base motion methods are employed to simulate the dynamics of the support plus branch combination to further evaluate the coupled base motion response analysis approach described in Appendix A. It should be noted that for the latter we first require the interface acceleration response time history to augment the coupled base motion response equation, given as Eq. 14 in the aforementioned appendix. This can be conveniently obtained by dynamic response analysis of the support only for the external excitations acting on the support system with no branch, or by application of Eq. 17 of Appendix A if the previous system had an old branch. Comparisons of the results obtained for the complete modal system and by the coupled base motion approach are made.

B. Description of Model

(1) Dynamic Model: Simplified structural models considered for the branch (or payload) and the support are shown in Figs. 9 and 10 respectively. The weight, AE and EI distribution assumed for these models are also shown in these figures. These

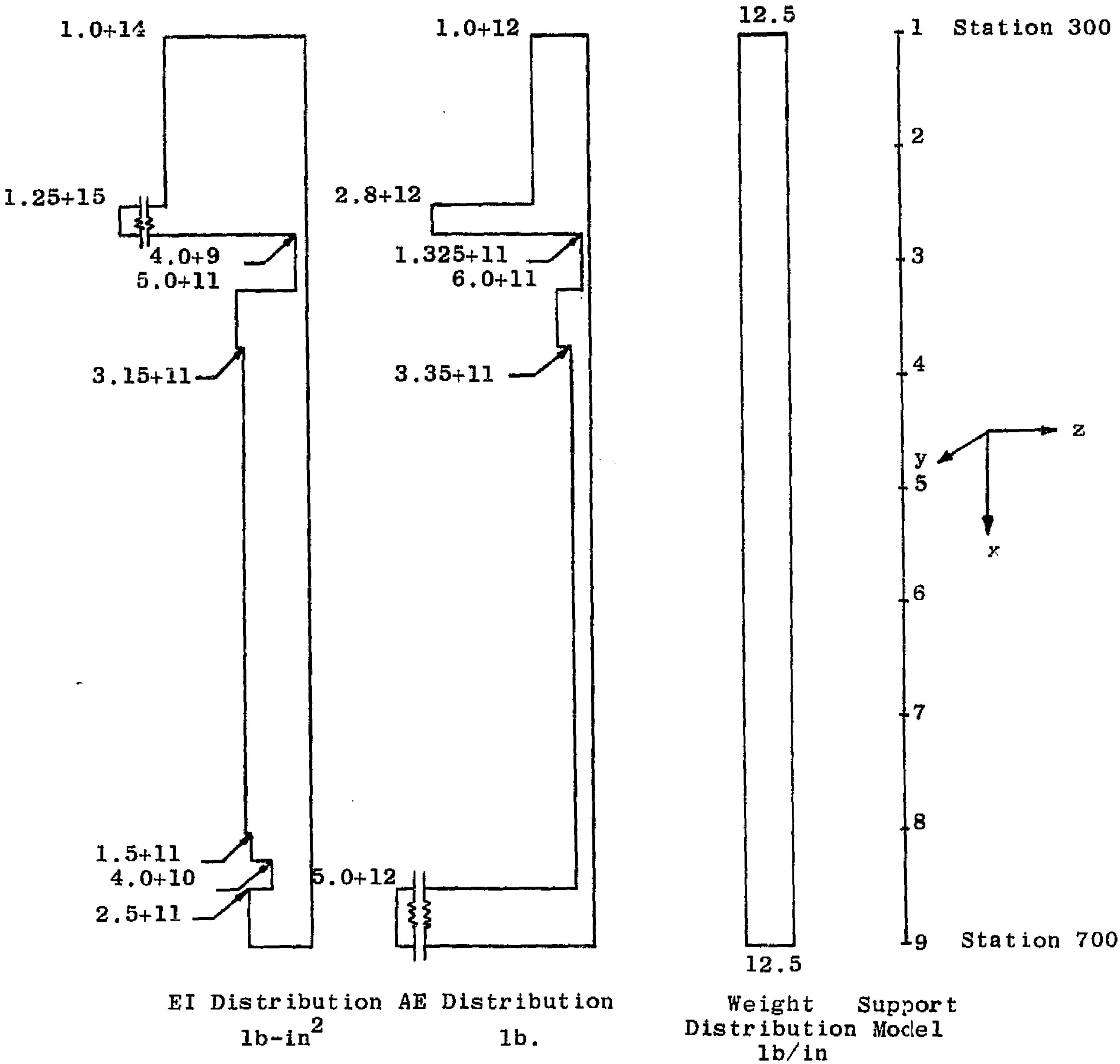


EI Distribution $lb\text{-}in^2$ | AE Distribution lb. | Weight Distribution lb/in | Branch Model

TOTAL WEIGHT = 6600 lbs

C.G. LOCATION: x CG (computed from z mass) = 109.1649 in. below Station 100
z CG (computed from x mass) = 1.051 in.

Fig. 9 Simplified Branch Model



TOTAL WEIGHT = 5000 lbs

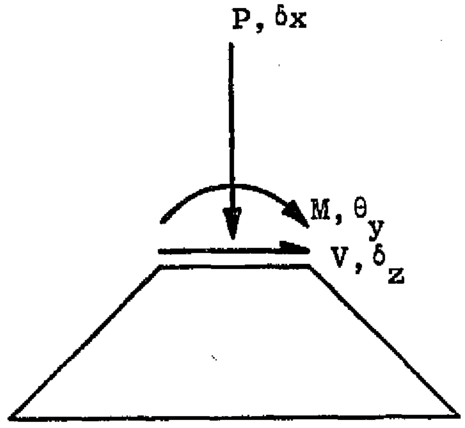
C.G. LOCATION: x CG (computed from z mass) = 200 in. below Station 300

z CG (computed from x mass) = 0.0 in.

Fig. 10 Simplified Support Model

distributions were basically obtained from similar data for the main axial beam of the HEAO-B Planar Model and Centaur Model. However, liberal simplifications were made with the aim of obtaining a simple demonstration problem. The total weight of the branch model is 6600 lbs and for the support it is 5000 lbs. An offset lumped mass of 3.63 slugs in the x and z plane is considered in the branch model, as shown in Fig. 9. This mass was transferred to joint 5 (Ref. Fig. 9) by the rigid arm branch coupling technique given in Appendix B. Thus only 9 joints are considered in the free-free branch system.

(2) Coupling Module: The Centaur Equipment Module is used to couple branch and support at the interface. The stiffness parameters of this module are given in Fig. 11. The coupling module was considered as an additional element of the cantilevered branch. Thus the cantilevered branch utilized in the base motion response analysis is as shown in Fig. 12.



$$\begin{Bmatrix} P \\ V \\ M \end{Bmatrix} = 10^8 \begin{bmatrix} 7.69231 & 0 & 0 \\ 0 & 8.42105 & 42.1053 \\ 0 & 42.1053 & 3240.83 \end{bmatrix} \begin{Bmatrix} \delta_x \\ \delta_z \\ \theta_y \end{Bmatrix}$$

Fig. 11 Coupling Module Stiffness Parameters

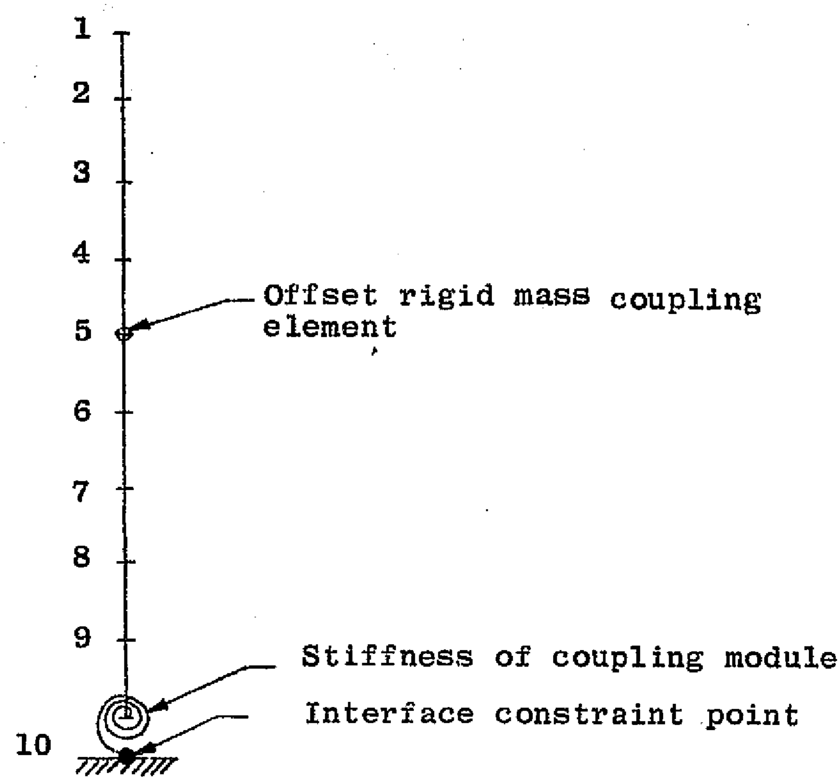


Fig. 12 Cantilevered Branch

(3) Coordinate System and Degrees of Freedom (DOF): Figure 13 shows the coordinate system and the DOF considered at each joint of the models.

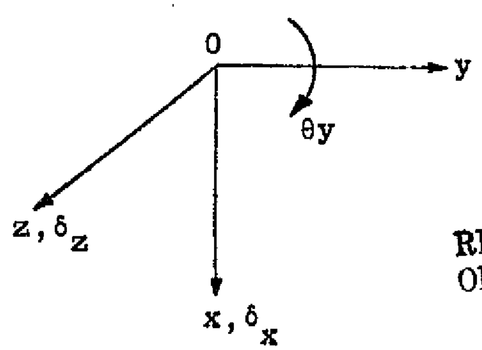


Fig. 13 Coordinate System and DOF Orientation

Tables III and IV contain the coordinates and the DOF numbering system of the cantilevered branch and support joints respectively.

TABLE III Cantilevered Branch Model Geometry

Joint Number	Coordinates			DOF Numbering System		
	x	y	z	x	z	θ_y
1	100.0	0.0	0.0	1	2	3
2	125.0	0.0	0.0	4	5	6
3	150.0	0.0	0.0	7	8	9
4	175.0	0.0	0.0	10	11	12
5	200.0	0.0	0.0	13	14	15
6	225.0	0.0	0.0	16	17	18
7	250.0	0.0	0.0	19	20	21
8	275.0	0.0	0.0	22	23	24
9	300.0	0.0	0.0	25	26	27
10	300.0	0.0	0.0	28	29	30

TABLE IV Support Model Geometry

Joint Number	Coordinates			DOF Numbering System		
	x	y	z	x	z	θ_y
1	300.0	0.0	0.0	1	2	3
2	350.0	0.0	0.0	4	5	6
3	400.0	0.0	0.0	7	8	9
4	450.0	0.0	0.0	10	11	12
5	500.0	0.0	0.0	13	14	15
6	550.0	0.0	0.0	16	17	18
7	600.0	0.0	0.0	19	20	21
8	650.0	0.0	0.0	22	23	24
9	700.0	0.0	0.0	25	26	27

(4) Vibration Analysis: FORMA Subroutine VIB3 was used to formulate the stiffness and mass matrices of the cantilevered branch, free-free support and integrated system. This subroutine also gives the mode shapes, frequencies, loads and modal coupling transformations and collapse matrix (i.e. [β]). Table V contains the modal frequencies analyzed for cantilevered branch, free-free support and integrated system.

TABLE V Vibration Analysis Frequencies*

	Frequency in Hertz		
Mode No.	Cantilevered Branch	Free-Free Support	Integrated System
1	5.84	0	0
2	28.08	0	0
3	54.31	0	0
4	79.40	28.14	7.78
5	126.68	111.82	27.85
6	153.15	274.04	44.73
7	250.86	403.93	80.13
8	314.23	608.17	99.28
9	337.72	970.60	115.50
10	475.06	1511.70	153.53

*N.B. Data for 10 modes only given here.

It was aimed to limit the frequencies to 100 Hertz so only 4 modes of the cantilevered branch system and 4 modes of the free-free support system were considered in the base motion response analysis. For the integrated system 8 modes were considered. These mode shapes are plotted in Figs. 14, 15 and 16.

Analysis for 8 cantilevered branch modes and 8 free-free support modes was also considered as a separate case.

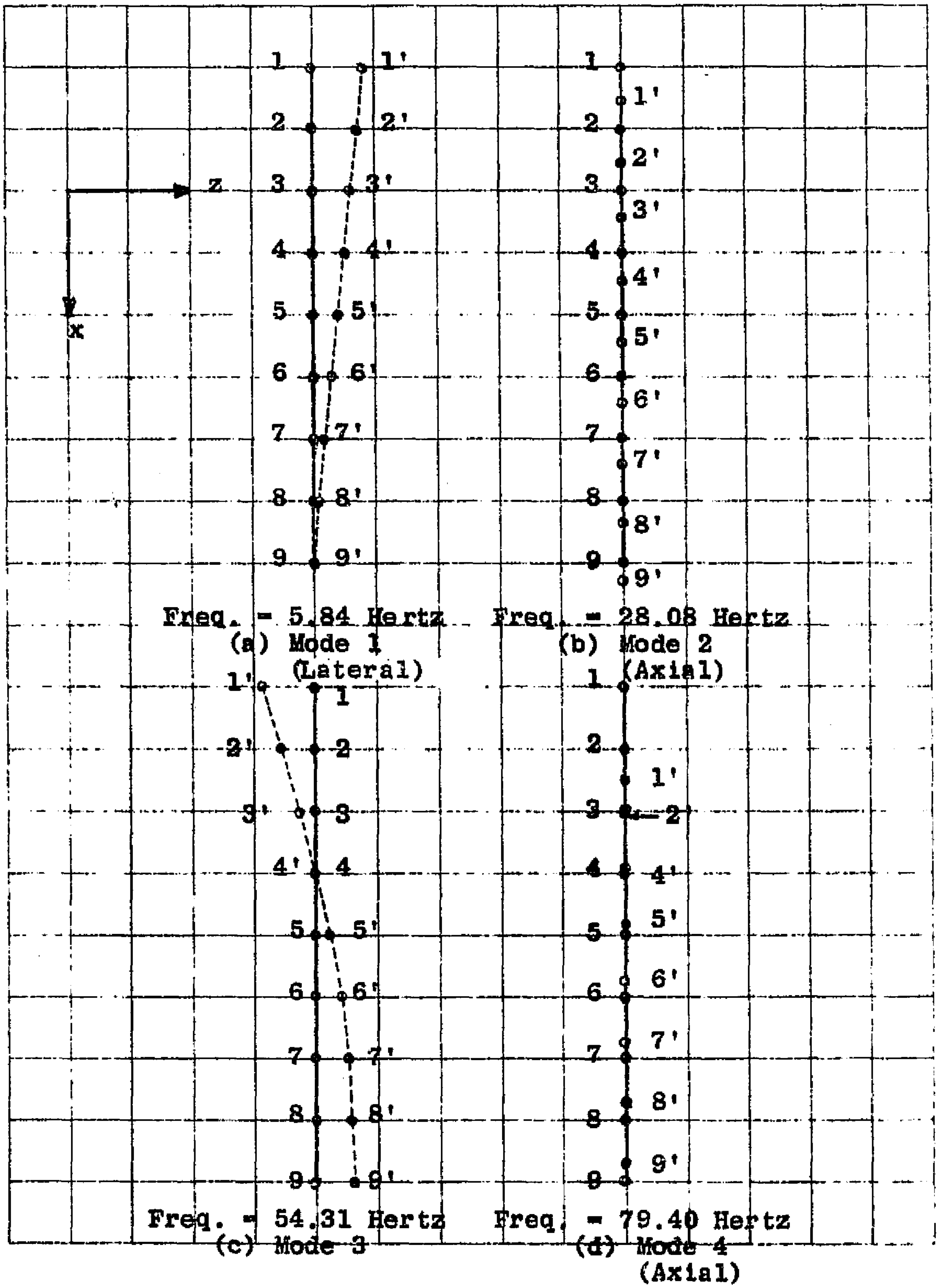


Fig. 14 Cantilevered Branch Modes

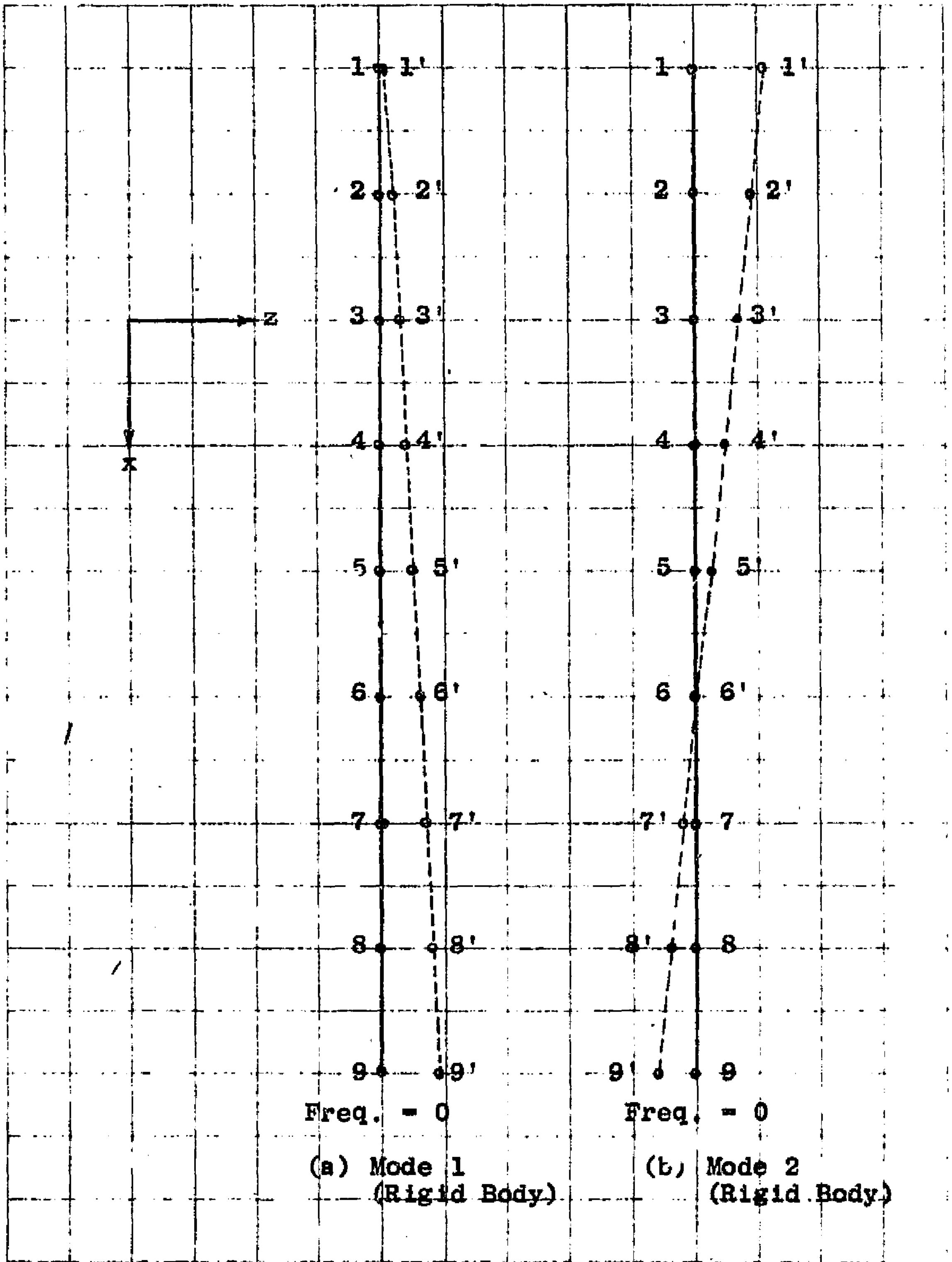
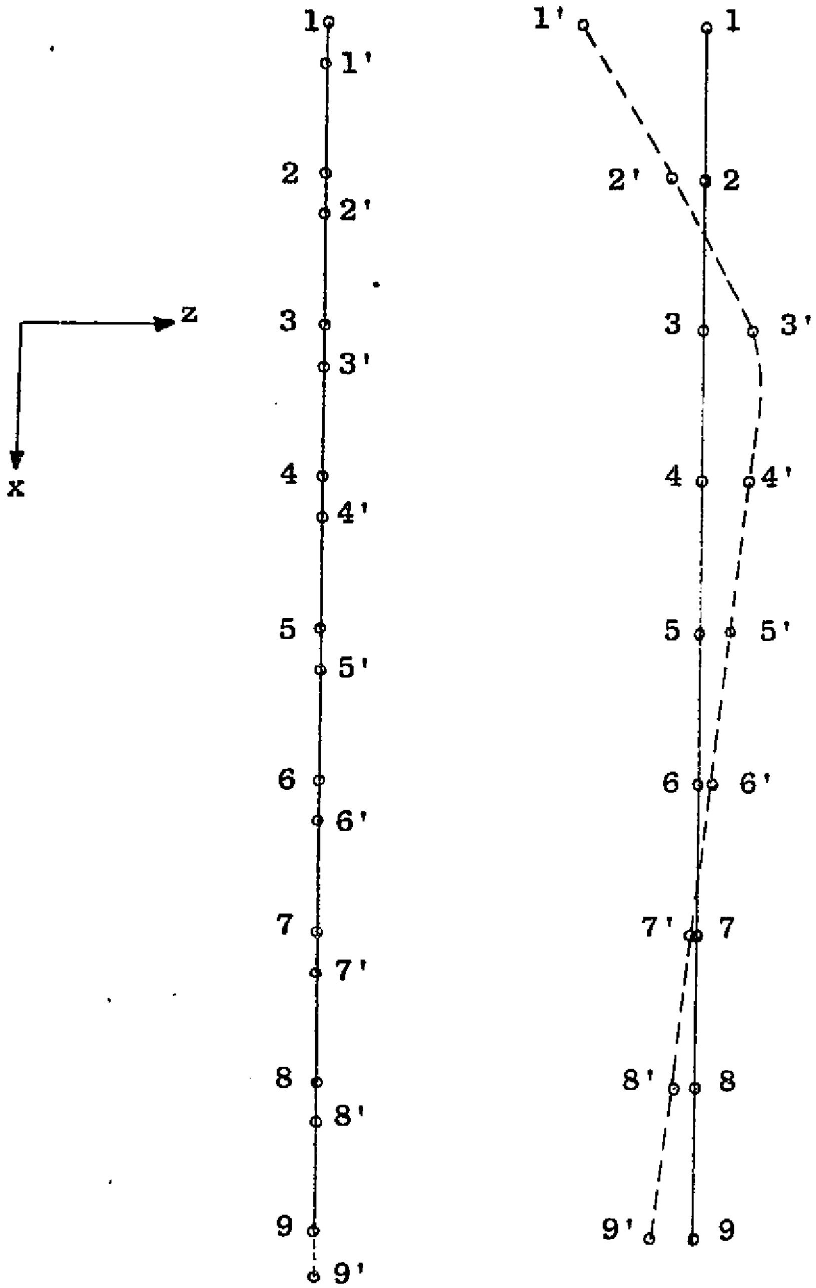


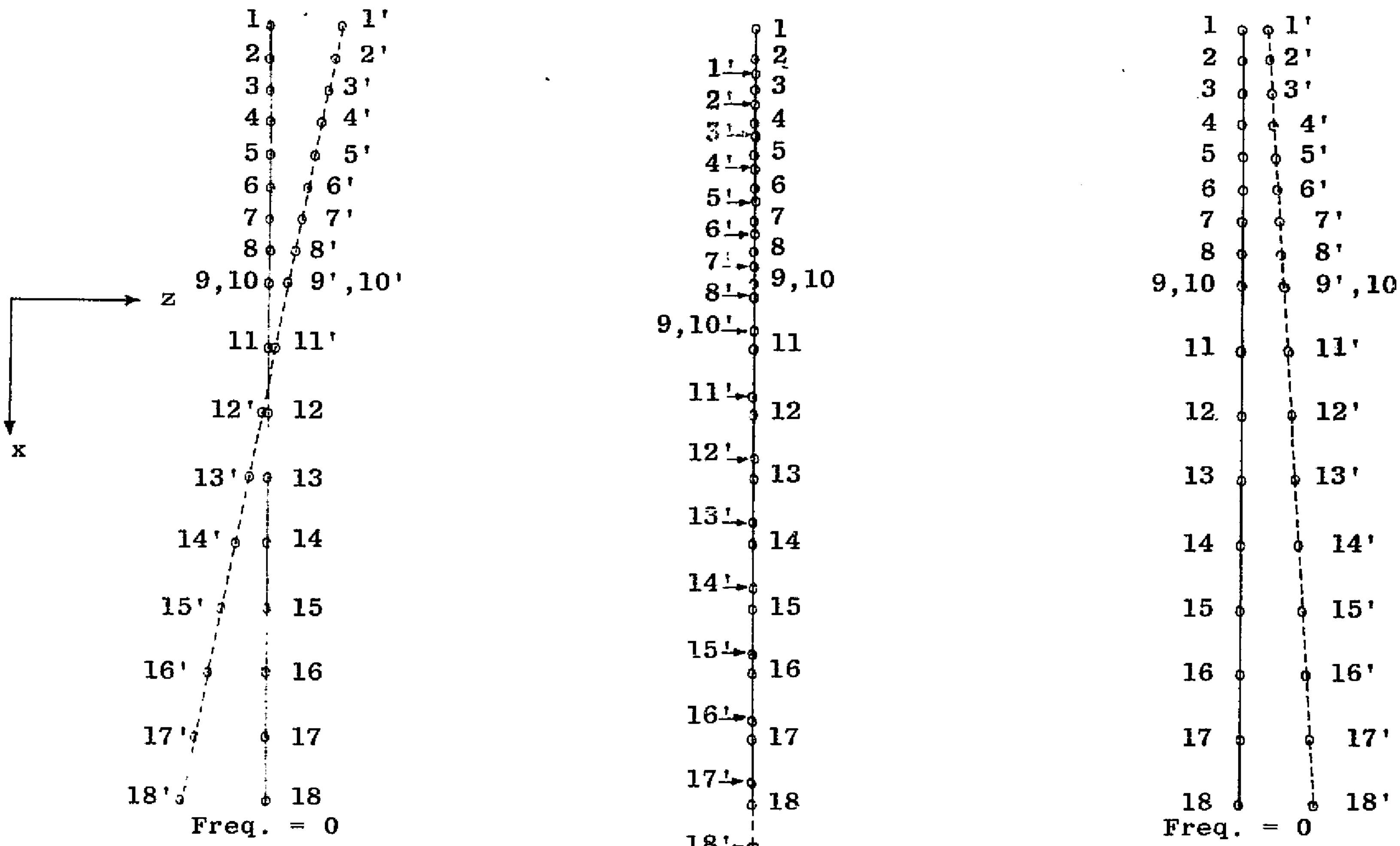
Fig. 15 Support Modes Free-Free



Freq. = 0
(c) Mode 3
(Rigid Body)

Freq. = 28.14 Hertz
(d) Mode 4

Fig. 15 (concluded) Support Modes Free-Free

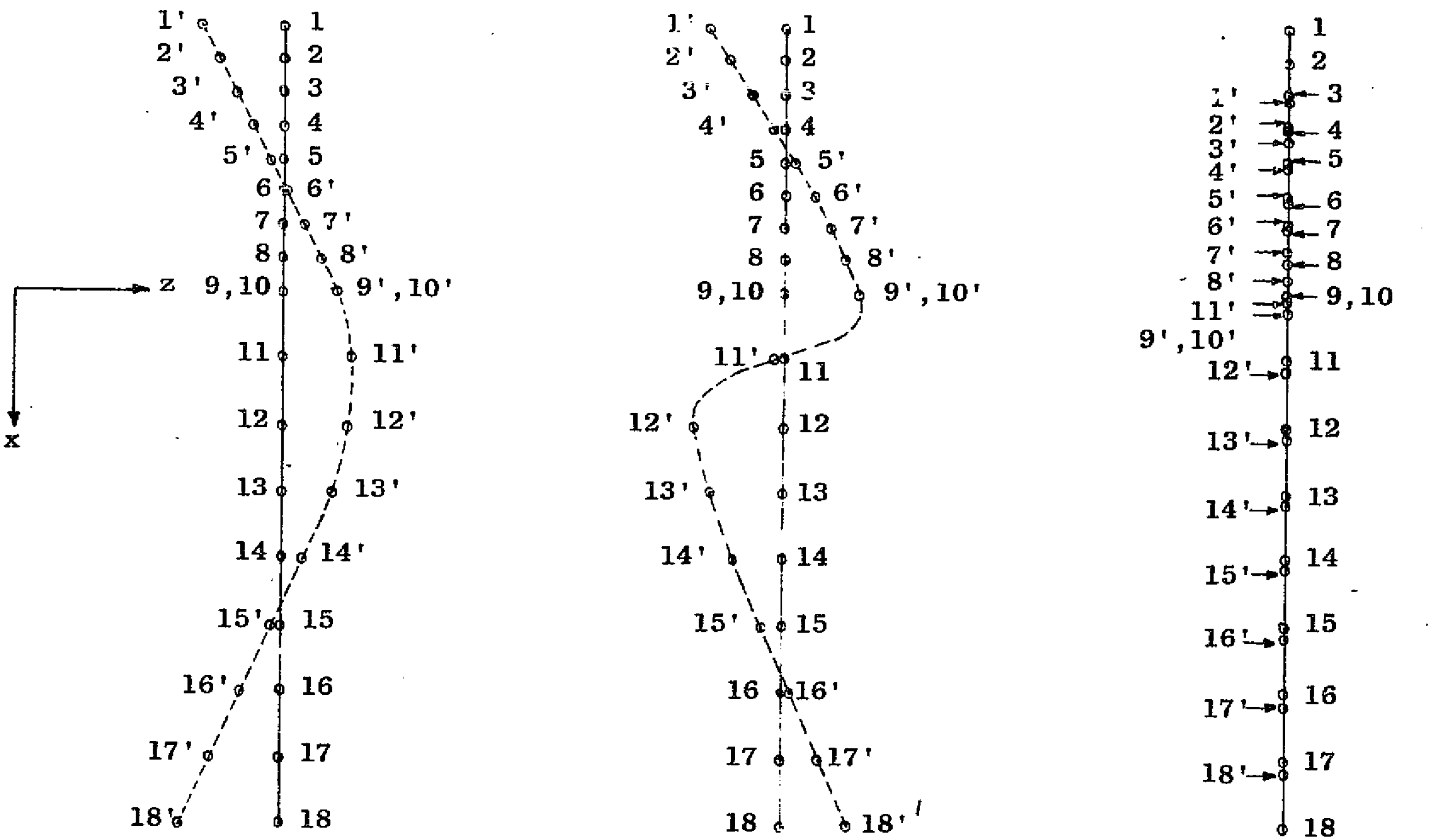


(a) Mode 1
(Rigid Body)

(b) Mode 2
(Rigid Body)

(c) Mode 3
(Rigid Body)

Fig. 16 Integrated System Modes

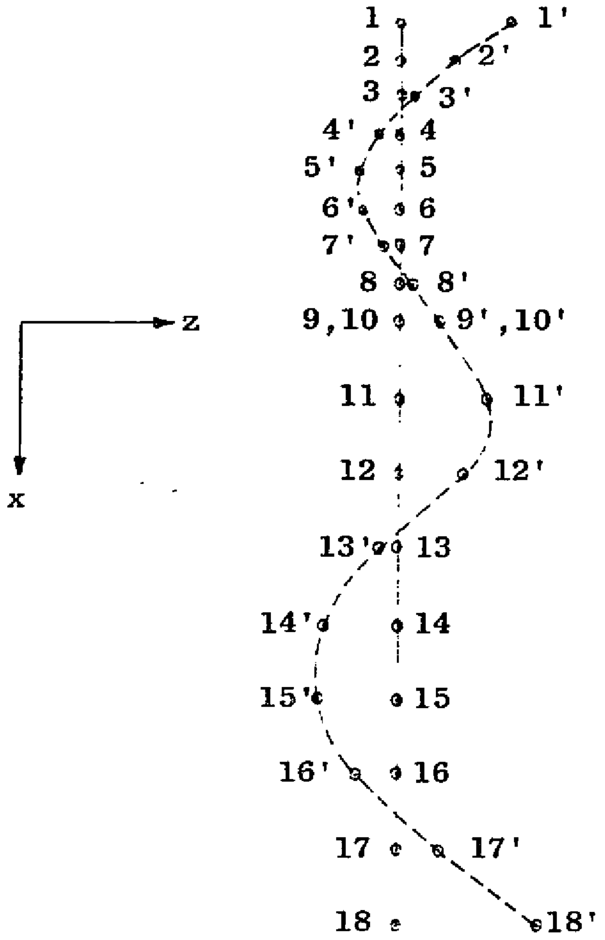


Freq. = 7.78 Hertz
(d) Mode 4

Freq. = 27.85 Hertz
(e) Mode 5

Freq. = 44.73 Hertz
(f) Mode 6

Fig. 16 (continued) Integrated System Modes



Freq. = 80.13 Hertz

(g) Mode 8

Fig. 16 (concluded) Integrated System Modes

(5) External Base Excitation: Two cases are considered.

Case A Axial excitation acting at joint 9 of support and along the negative direction of the 25th DOF. The variation of this base excitation with time is shown in Fig. 17.

Case B Axial and moment excitations acting at joint 9 of support and along the negative directions of 25th and 27th DOF respectively. The variations of these base excitations with time are shown in Fig. 18.

(6) Interface Response History: The support system was analyzed for both Case A and Case B excitations. The response equation used is:

$$\{\ddot{\xi}\}_S + \lceil 2\rho\omega \rfloor_S\{\dot{\xi}\} + \lceil \omega^2 \rfloor_S\{\xi\} = [\varphi]_S^T\{F(t)\}$$

where $\{\ddot{\xi}\}_S$, $\{\dot{\xi}\}_S$, and $\{\xi\}_S$ = Modal accelerations, velocities and displacements vectors for support

$\lceil \omega^2 \rfloor_S$ = Circular frequency squared matrix for support

$[\varphi]_S$ = Support mode shapes (free-free)

ρ_S = Damping coefficient for support

Using the data obtained from FORMA Subroutine VIB3 for $[\varphi]_S$ and $\lceil \omega^2 \rfloor_S$ of the support and taking the damping coefficient ρ_S = 0.01, FORMA Subroutine TR3 was used to formulate discrete interface acceleration history. 5 modes of the support were considered in this analysis.

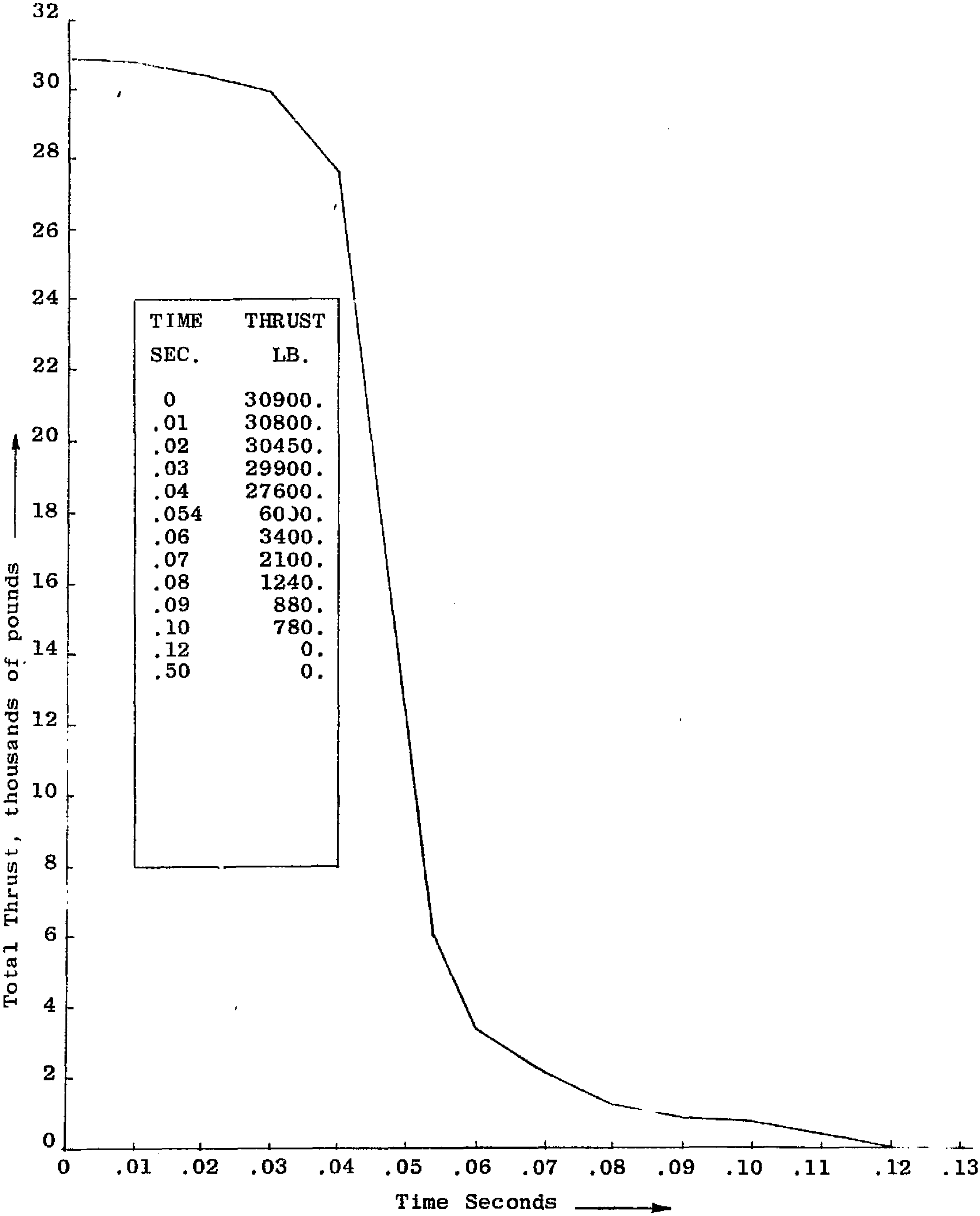
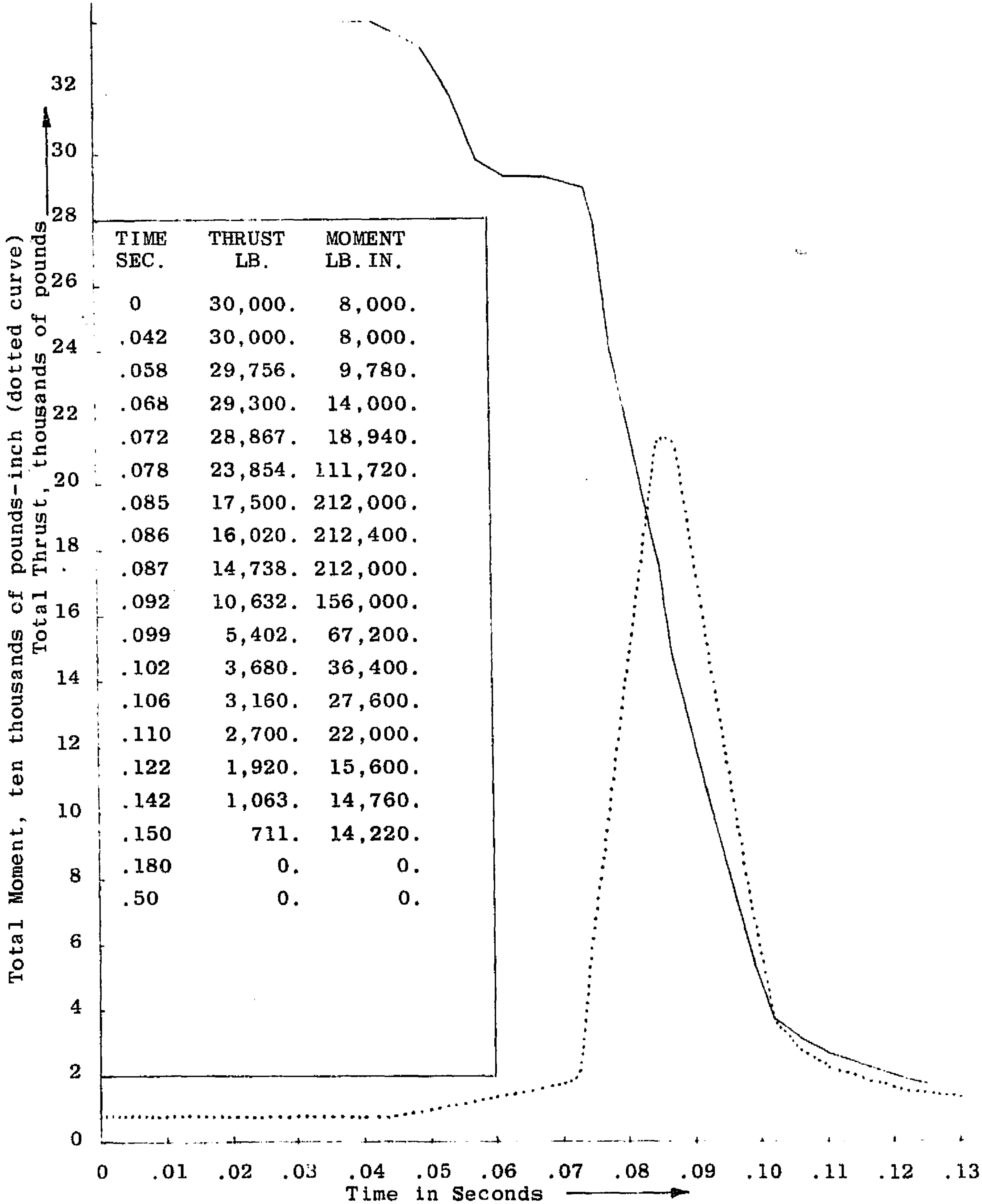


Fig. 17 Case A - External Base Excitation



TIME SEC.	THRUST LB.	MOMENT LB. IN.
0	30,000.	8,000.
.042	30,000.	8,000.
.058	29,756.	9,780.
.068	29,300.	14,000.
.072	28,867.	18,940.
.078	23,854.	111,720.
.085	17,500.	212,000.
.086	16,020.	212,400.
.087	14,738.	212,000.
.092	10,632.	156,000.
.099	5,402.	67,200.
.102	3,680.	36,400.
.106	3,160.	27,600.
.110	2,700.	22,000.
.122	1,920.	15,600.
.142	1,063.	14,760.
.150	711.	14,220.
.180	0.	0.
.50	0.	0.

Fig. 18 Case B - External Base Excitations

C. Results and Observations

Feed-Back Base Drive Program (see Section III) was used to formulate and solve the coupled base motion response equation. Some of the results obtained for discrete accelerations of the branch for Cases A and B are shown in Figs. 19 through 23. All these curves are drawn for results obtained considering 4 modes of support and 4 modes of cantilevered branch (4+4). Results obtained for the integrated system, considering 8 modes are also indicated in these Figures. As can be seen from these figures the results for the 4+4 modes, for both cases, agree very closely with the integrated analysis results. Results were also checked for 8+8 nodes (i.e. 8 support modes + 8 branch modes) and it was found that these fall between the curves for 4+4 modes and integrated system. It should be noted that in the 4+4 cases, no axial elastic support system mode was present (i.e. rigid body for x only), thus all elastic interface motion (axially) resulted from the feed back coupling. The curves are drawn for some of the degrees of freedom only, in both cases. Similar agreement of the results was found for other degrees of freedom. In conclusion, we note that the method gave excellent results for a rather detailed model even with considerable modal truncation. Comparison of results from base motion methods with those obtained from the total integrated system analysis constitute sufficient arguments as to the validity of the coupled base motion methodology.

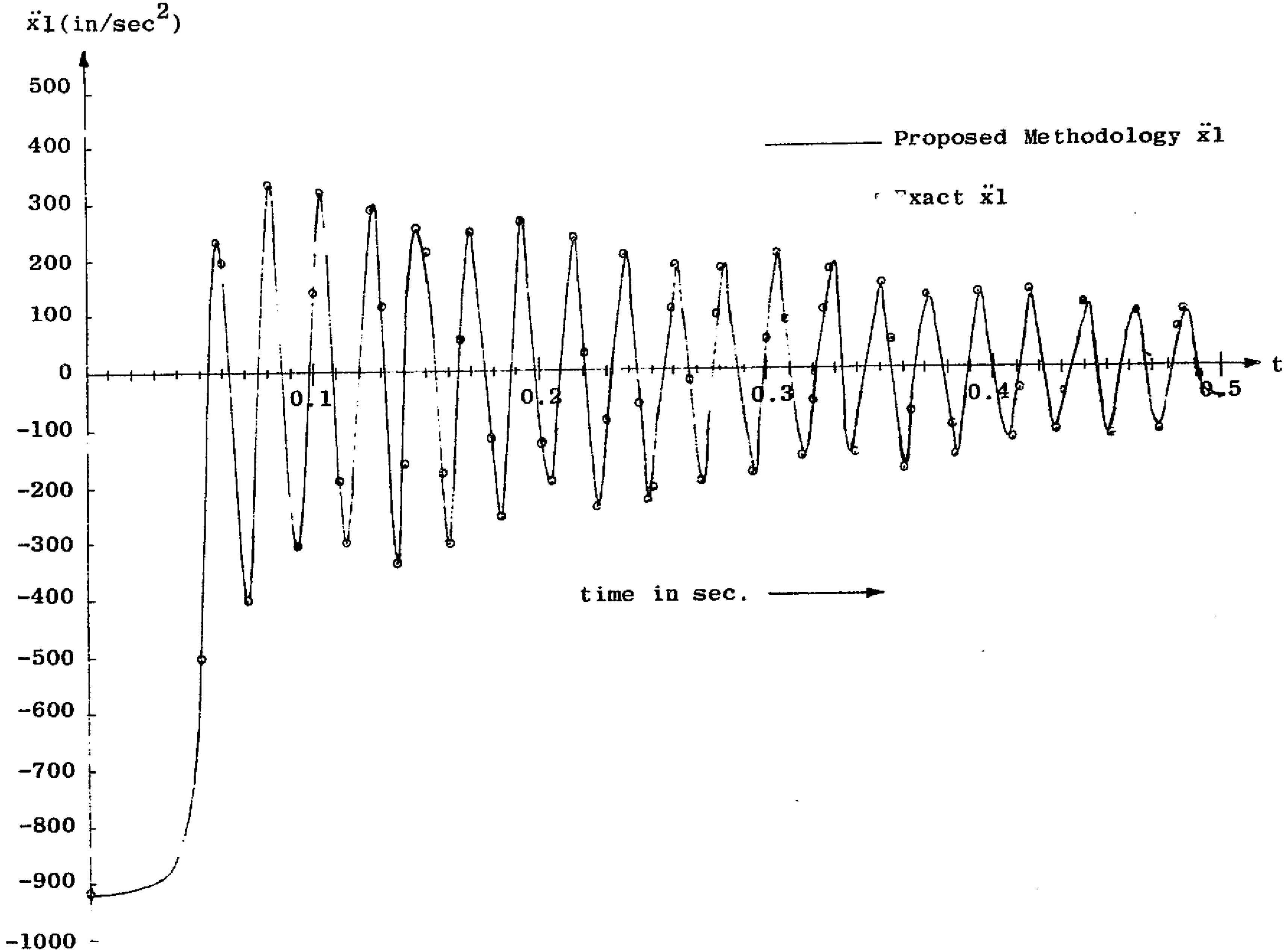


Fig. 19 Case A - ẍ1 vs. Time

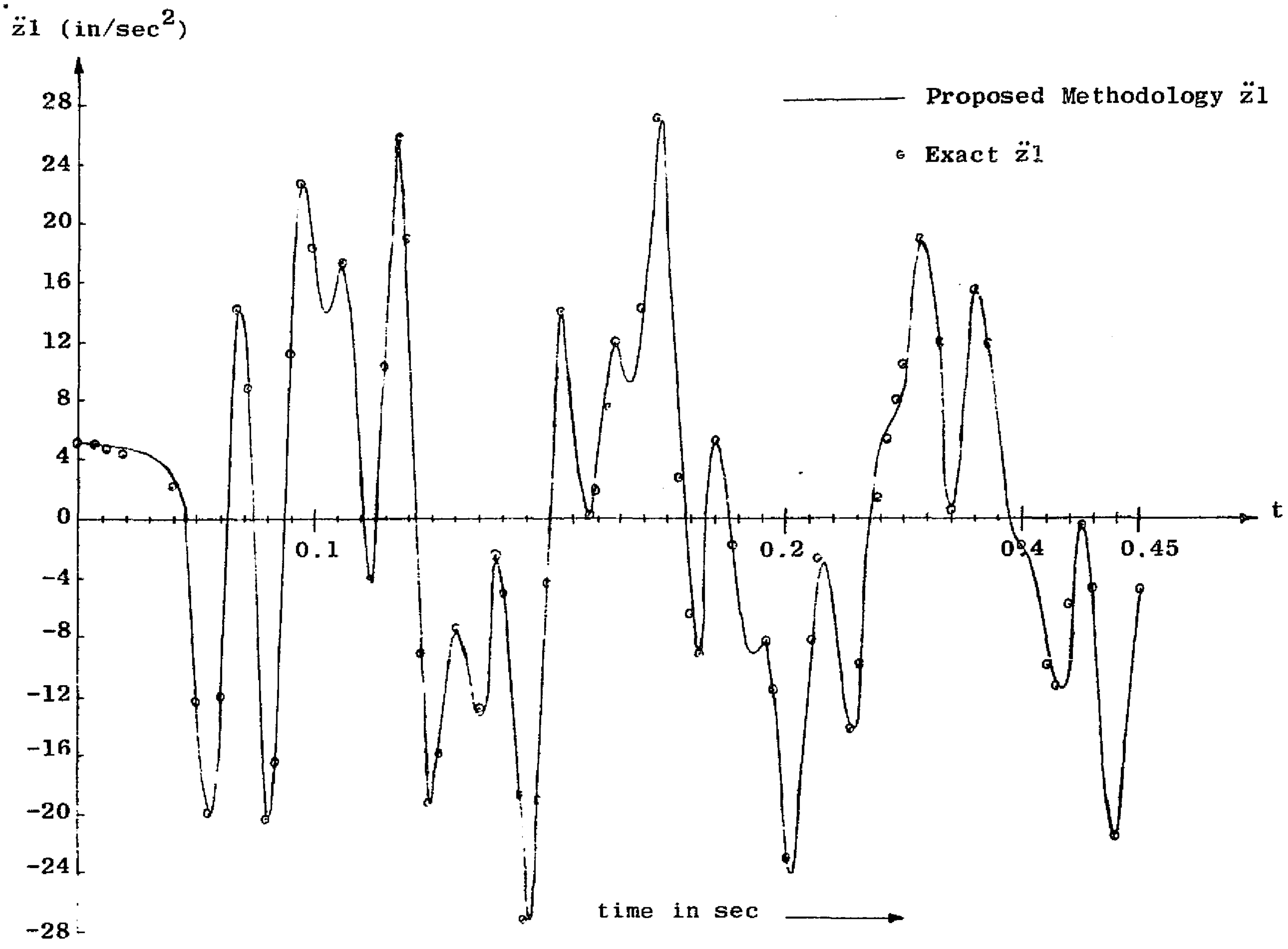


Fig. 20 Case A - $\ddot{z}1$ vs. Time

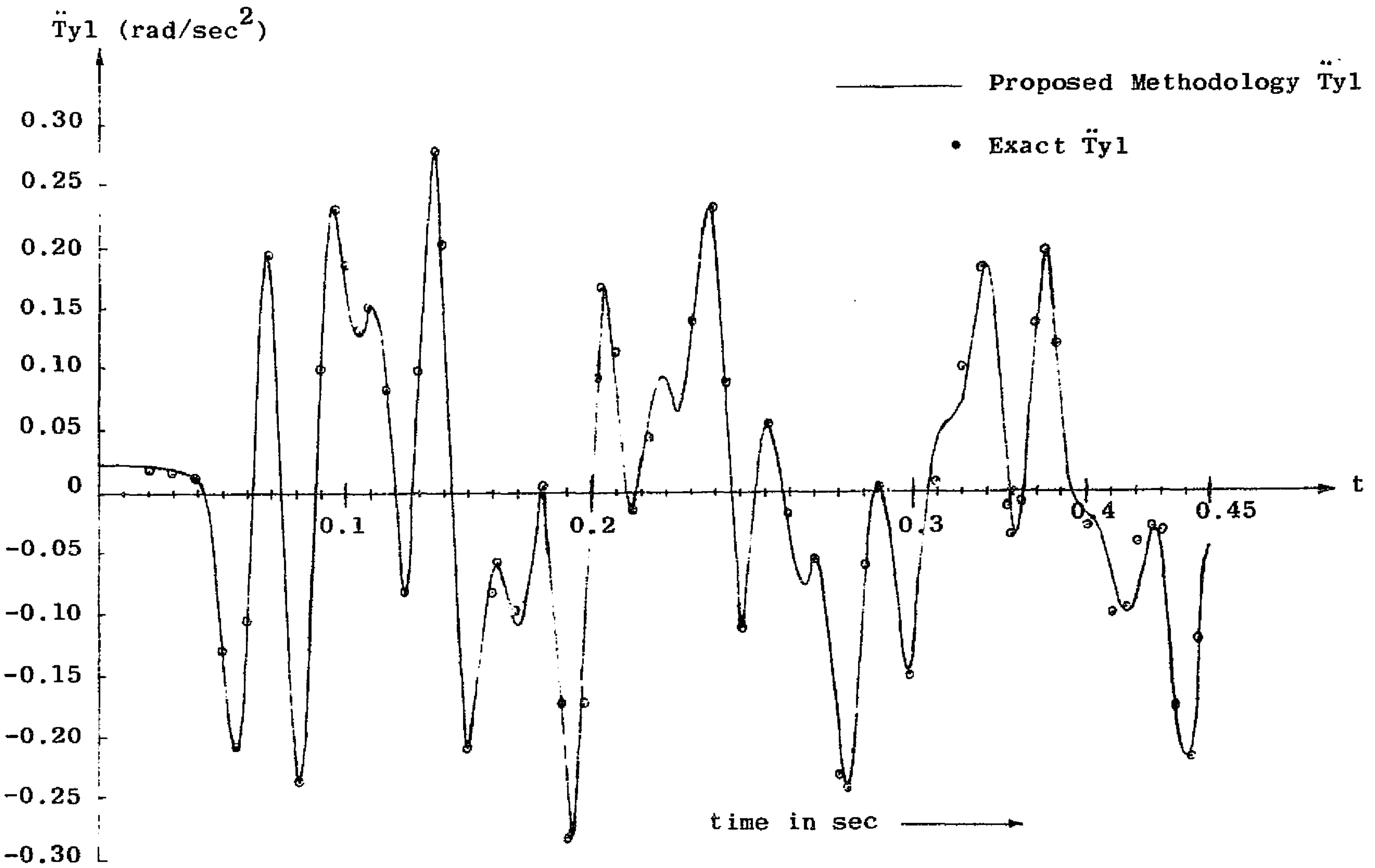


Fig. 21 Case A - $\ddot{T}y1$ vs. Time

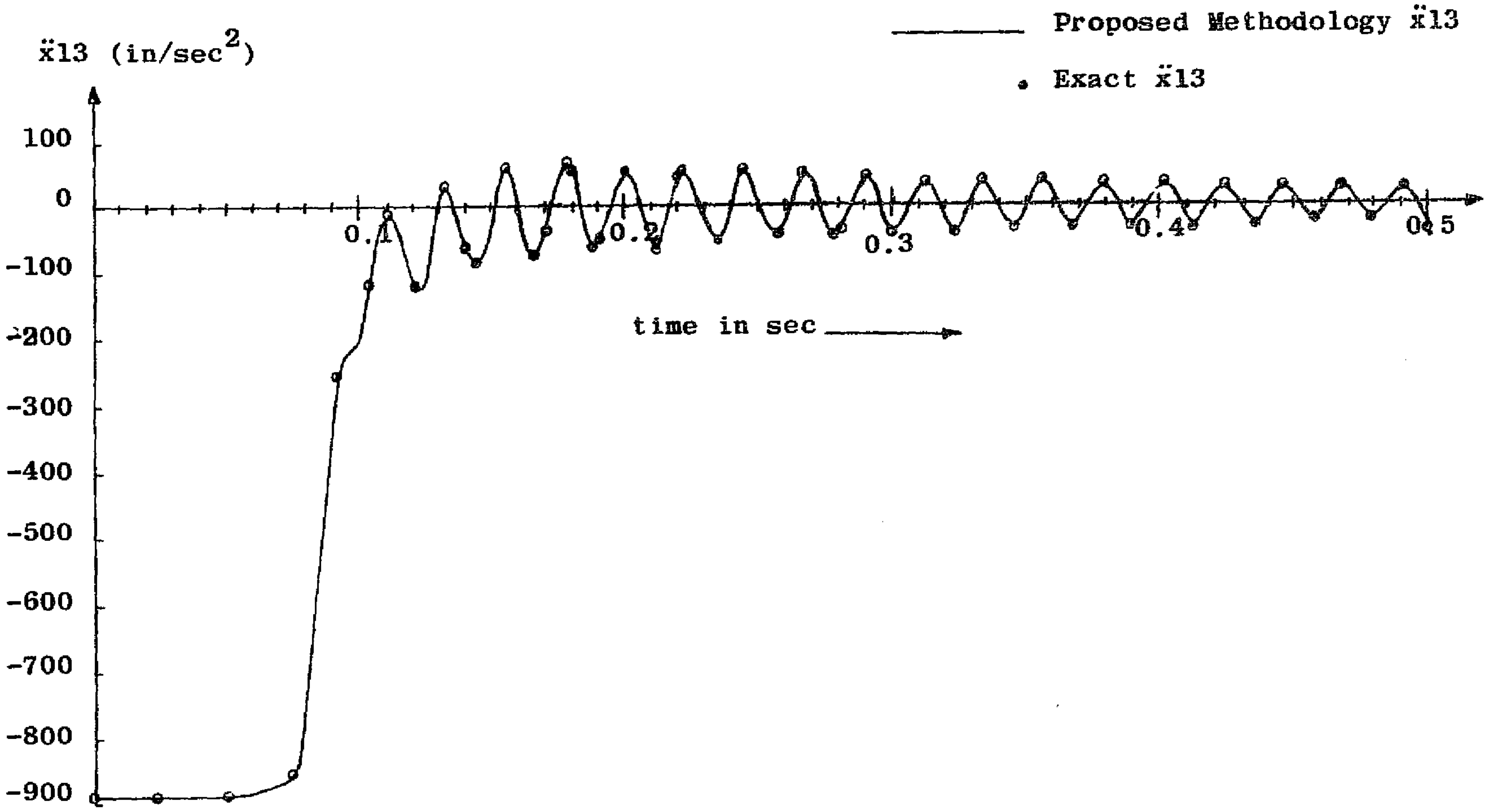


Fig. 22 Case B - ẍ13 vs. Time

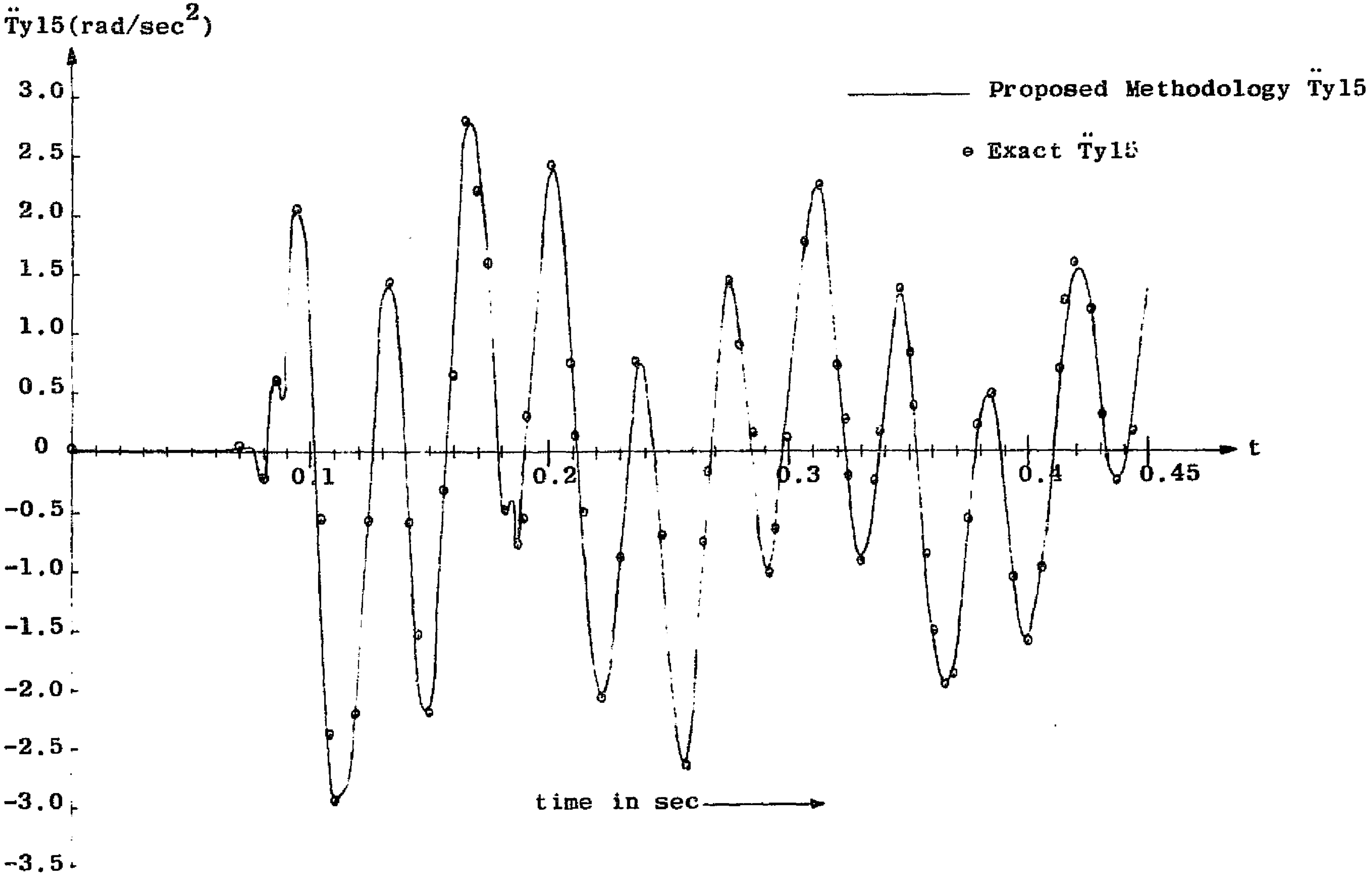


Fig. 23 Case B - $\ddot{T}y15$ vs. Time

III. Program Code

A. Program Logic

Feed-Back Base Drive Program or FBBD is a program meant to formulate the coefficient matrices of the coupled base motion response equation and calculate discrete accelerations and internal loads of the branch system. The programming of FBBD is based on a collection of FORTRAN matrix algebra routines called FORMA (Ref. 7), as developed by the Dynamics and Load Section of the Martin Marietta Corporation, Denver Division. The user of this program is assumed to have a working knowledge of FORMA and its associated terminology.

The program FBBD is divided into four main blocks, viz.

(1) Block I: Formulation of the coefficient matrices of the coupled base motion response equation. The coupled base motion response equation used is derived as Eq. (14) in Appendix A and reproduced below.

$$
\left[\begin{array}{c:c}
\lceil I \rfloor & [\varphi_C]_B^T [M]_B [\beta] [\varphi']_S \\
\hdashline
-[\varphi]_S^T [TB] & \lceil I \rfloor - [\varphi]_S^T [TS]
\end{array}\right]
\left\{\begin{array}{c} \ddot{q}_B \\ \hdashline \ddot{q}_S \end{array}\right\}
+
\left[\begin{array}{c:c}
\lceil 2\rho\omega \rfloor_B & 0 \\
\hdashline
0 & \lceil 2\rho\omega \rfloor_S
\end{array}\right]
\left\{\begin{array}{c} \dot{q}_B \\ \hdashline \dot{q}_S \end{array}\right\}
$$

$$
+
\left[\begin{array}{c:c}
\lceil \omega^2 \rfloor_B & 0 \\
\hdashline
0 & \lceil \omega^2 \rfloor_S
\end{array}\right]
\left\{\begin{array}{c} q_B \\ \hdashline q_S \end{array}\right\}
=
\left[\begin{array}{c}
-[\varphi_C]_B^T [M]_B [\beta] \\
\hdashline
[\varphi_S]^T [TI]
\end{array}\right]
\left\{\ddot{\delta}_{ES}(t)\right\}
\tag{36}
$$

where

$[\varphi_C]_B$ = Mode shapes matrix of cantilevered branch

$[M]_B$ = Mass matrix of cantilevered branch

$[\varphi']_S$ = Support free-free interface modes matrix

$\lceil\omega\rfloor_B$ = Branch circular frequency diagonal matrix

$\lceil\omega\rfloor_S$ = Support circular frequency diagonal matrix

$\lceil\rho\rfloor_B$ = Branch damping coefficient matrix

$\lceil\rho\rfloor_S$ = Support damping coefficient matrix

$[\beta]$ = Collapse transformation matrix

$\{\ddot{q}_B\},\{\dot{q}_B\},\{q_B\}$ = Modal accelerations, velocities and displacements of cantilevered branch

$\{\ddot{q}_S\},\{\dot{q}_S\},\{q_S\}$ = Modal accelerations, velocities and displacements of support

ρ = Ratio of viscous damping and critical viscous damping (i.e. C/C_R)

$[TB] = -[\Gamma][M]_B[\varphi_C]_B = -[\beta]^T[M]_B[\varphi_C]_B$

$\therefore [\Gamma]$ = The interface loads transformation matrix which generalizes branch force to the interface

$= [\beta]^T$

$[TS] = -[\Gamma][M]_B[\beta][\varphi']_S = -[\beta]^T[M]_B[\beta][\varphi']_S$

$[TI] = -[\Gamma][M]_B[\beta] = -[\beta]^T[M]_B[\beta]$

and $\{\ddot{\delta}_{FE}(t)\}$ = Interface acceleration distribution

In the computer program FBBD Eq. (36) is defined as

$$\left[\begin{array}{c|c} A(1,1) & A(1,2) \\ \hline A(2,1) & A(2,2) \end{array}\right]\left\{\begin{array}{c} \ddot{q}_B \\ \hline \ddot{q}_S \end{array}\right\}+\left[\begin{array}{c|c} B(1,1) & 0 \\ \hline 0 & B(2,2) \end{array}\right]\left\{\begin{array}{c} \dot{q}_B \\ \hline \dot{q}_S \end{array}\right\}+\left[\begin{array}{c|c} C(1,1) & 0 \\ \hline 0 & C(2,2) \end{array}\right]\left\{\begin{array}{c} q_B \\ q_S \end{array}\right\}$$

$$= \left[\begin{array}{c} D(1) \\ \hline D(2) \end{array}\right]\{\ddot{\delta}_{ES}(t)\} \qquad (37)$$

or $[A]\{\ddot{q}\} + [B]\{\dot{q}\} + [C]\{q\} = [D]\{\ddot{\delta}_{ES}(t)\}$

where

$$
\begin{aligned}
A(1,1) &= \lceil I \rfloor \\
A(1,2) &= [\varphi_C]_B^T[M]_B[\beta][\varphi']_S \\
A(2,1) &= -[\varphi'_S]^T[TB] = -[\varphi']_S^T\left(-[\beta]^T[M]_B[\varphi_C]_B\right) \\
&= [\varphi']_S^T[\beta]^T[M]_B[\varphi_C]_B \\
&= \left(A(1,2)\right)^T \\
A(2,2) &= [I] - [\varphi']_S^T[TS] = [I] - [\varphi']_S^T\left(-[\beta]^T[M]_B[\beta][\varphi']_S\right) \\
&= [I] + [\varphi']_S^T[\beta]^T[M]_B[\beta][\varphi']_S \qquad (38) \\
B(1,1) &= \lceil 2\rho\omega \rfloor_B \\
B(2,2) &= \lceil 2\rho\omega \rfloor_S \\
C(1,1) &= \lceil \omega^2 \rfloor_B \\
C(2,2) &= \lceil \omega^2 \rfloor_S \\
D(1) &= -[\varphi_C]_B^T[M]_B[\beta] \\
\text{and} \quad D(2) &= [\varphi']_S^T[TI] = -[\varphi']_S^T[\beta]^T[M]_b[\beta]
\end{aligned}
$$

The matrices $[\varphi_C]_B$, $[\varphi']_S$, $[M]_B$, $\lceil\omega^2\rfloor_B$, $\lceil\omega^2\rfloor_S$ and $[\beta]$ are first generated using FORMA subroutine VIB3 and are input in program FBBD. Data for damping coefficients, i.e. $\lceil\rho\rfloor_B$ and $\lceil\rho\rfloor_S$ are also input in this section. Using these matrices, the coefficient matrices [A], [B], [C] and [D] are formulated as per the matrix operations defined in Eq. (38).

(2) <u>Block II</u>: Solution of coupled base motion response equation. Having formulated the coefficient matrices in Block I, Eq. (37) is solved for modal accelerations using subroutine TR1FBD (i.e. Transient Response for

Feed-Back Base Drive). This subroutine is basically a response routine to solve a second order differential equation by fourth order Runge-Kutta (Gill modification) numerical integration. The initial conditions for velocity and displacements have to be established in order to solve this differential equation. In our case, initial velocities at each DOF can be taken as zero, but the initial elastic displacements have to be computed using the data for external forces at initial time. The procedure to do this is explained in the next paragraph.

The modal displacement vector $\{q\} = \begin{Bmatrix} q_B \\ \hline q_S \end{Bmatrix}$ can be repartitioned, by placing the displacements connected with rigid body modes on top partition and the remaining elastic displacements below it, i.e.

$$\{q\} = \begin{Bmatrix} q_R \\ \hline q_e \end{Bmatrix} \tag{39}$$

But

$$\{q_R\}_{t=0} = \{0\}$$

and

$$\begin{Bmatrix} \dot{q}_R \\ \hline \dot{q}_e \end{Bmatrix}_{t=0} = \{0\} \tag{40}$$

In view of Eqs. (39) and (40), Eq. (37) can be rewritten in partition form for t = 0, as

$$\left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] \left\{\begin{array}{c} \ddot{q}_R \\ \hline \ddot{q}_e \end{array}\right\}_{t=0} + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & K_{22} \end{array}\right] \left\{\begin{array}{c} q_R \\ \hline q_e \end{array}\right\}_{t=0} = \left[\begin{array}{c} D_R \\ \hline D_E \end{array}\right] \{\ddot{\delta}_{FE}(t)\}_{t=0} \quad (41)$$

The first of these equations is

$$[M_{11}]\{\ddot{q}_R\}_{t=0} + [M_{12}]\{\ddot{q}_e\}_{t=0} = [D_R]\{\ddot{\delta}_{FE}(t)\}_{t=0}$$

But $\{\ddot{q}_e\}_{t=0} = \{0\}$

$$\therefore \quad \{\ddot{q}_R\}_{t=0} = [M_{11}]^{-1}[D_R]\{\ddot{\delta}_{FE}(t)\}_{t=0} \quad (42)$$

The second equation of (41) thus gives

$$[M_{21}]\{\ddot{q}_R\}_{t=0} + [K_{22}]\{q_e\}_{t=0} = [D_E]\{\ddot{\delta}_{FE}(t)\}_{t=0}$$

$$\therefore \quad \{\ddot{q}_e\}_{t=0} = [K_{22}]^{-1}\Big([D_E]\{\ddot{\delta}_{FE}(t)\}_{t=0} - [M_{21}]\{\ddot{q}_R\}_{t=0}\Big)$$

In view of Eq. (42) this reduces to

$$\{\ddot{q}_e\}_{t=0} = [K_{22}]^{-1}\Big([D_E]\{\ddot{\delta}_{FE}(t)\}_{t=0} - [M_{21}][M_{11}]^{-1}[D_R]\{\ddot{\delta}_{FE}(t)\}_{t=0}\Big)$$

$$= [K_{22}]^{-1}\Big([D_E] - [M_{21}][M_{11}]^{-1}[D_R]\Big)\{\ddot{\delta}_{FE}(t)\}_{t=0} \quad (43)$$

So in view of Eqs. (42) and (43) we can write

$$\left\{\begin{array}{c} \ddot{q}_R \\ \hline \ddot{q}_e \end{array}\right\}_{t=0} = \left[\begin{array}{c|c} M_{11}^{-1} & 0 \\ \hline -K_{22}^{-1}M_{21}M_{11}^{-1} & K_{22}^{-1} \end{array}\right] \left[\begin{array}{c} D_R \\ \hline D_E \end{array}\right] \{\ddot{\delta}_{FE}(t)\}_{t=0}$$

$$= \left[\begin{array}{c|c} M_{11} & 0 \\ \hline M_{21} & K_{22} \end{array}\right]^{-1} \left[\begin{array}{c} D_R \\ \hline D_E \end{array}\right] \{\ddot{\delta}_{FE}(t)\}_{t=0} \quad (44)$$

Eq. (44) can thus be used to find the displacements

at $t = 0$, i.e. the bottom partition of the desired initial elastic modal deflections.

$$\{q\}_{t=0} = \left\{\begin{array}{c} q_R \\ \hdashline q_e \end{array}\right\}_{t=0} \Longrightarrow \left\{\begin{array}{c} 0 \\ \hdashline q_e \end{array}\right\}_{t=0} \tag{45}$$

In subroutine TRI‾ED the initial displacements are computed using the aforementioned approach.

(3) Block III: Calculation of discrete accelerations of branch.

The discrete acceleration of the branch can be found using Eq. (2) of Appendix A, i.e.

$$\{\ddot{\delta}_B(t)\} = [\beta]\{\ddot{\delta}_S(t)\} + [\varphi_C]_B[\ddot{q}_B] \tag{46}$$

where $\{\ddot{\delta}(t)_S\}$ = Discrete acceleration of support

$$= [\varphi']_S\{\ddot{\delta}_{FE}(t)\} + [\varphi']_S\{\ddot{q}_S\}$$

Substituting this in Eq. (46) yields

$$\{\ddot{\delta}_B(t)\} = [\beta][\varphi']_S\{\ddot{\delta}_{FE}(t)\} + \left[\begin{array}{c:c} [\varphi_C]_B & 0 \\ \hdashline 0 & [\beta][\varphi']_S \end{array}\right] \left\{\begin{array}{c} \ddot{q}_B \\ \ddot{q}_S \end{array}\right\} \tag{47}$$

Having found the modal accelerations $\left\{\begin{array}{c} \ddot{q}_B \\ \hdashline \ddot{q}_S \end{array}\right\}$ from the solution of the base motion response equation in Block II and knowing the interface acceleration history $\{\ddot{\delta}_{FE}(t)\}$, Eq. (47) is used to find discrete accelerations $\{\ddot{\delta}_B(t)\}$ of the branch. This is done in subroutine TACCBD (i.e. Total Acceleration of Base Drive).

(4) Block IV: Calculation of internal loads. The internal loads of the branch can be expressed as

$$\{L_B\} = [LCT][-M]_B\{\ddot{\delta}_B\} \tag{48}$$

where $\{L_B\}$ = Internal loads vector of branch

$[LCT]$ = Loads coefficient transformation matrix

$[M]_B$ = System mass matrix of branch

and $\{\ddot{\delta}_B\}$ = Discrete accelerations of branch, obtained from Block III

The loads coefficient transformation matrix $[LCT]$ and mass matrix $[M]_B$ can be obtained from FORMA subroutine VIB3 or by any other conventional structural dynamics modal analysis package. In general a separate $[LCT]$ for axial beam, bending beam (both shear and moment) and any spring stiffness elements are identified. The product $[LCT][-M]_B$, thus gives the load transformation matrix which is discrete acceleration type. Based on this approach subroutine ACCLD calculates the internal loads in branch.

```
C      ****************************************************************
C
C      B. DATA STREAM FLOW
C
C      ****************************************************************
       DATA  K1 / K2 / K3 /
      *        70 / 15 / 40 /
C         5 CALL START
C          READ(5,1001) NCPB,NRO,NCO,NU1,NU2,NRTAPE,NXTAPE,NPRT,NWT,IFMC
C                                                      FORMAT         (16I5)
C          READ(5,1002) IFINIT,TAPEID                  FORMAT         (12A6)
C          READ(5,1003) STARTT,DELTAT,ENDT             FORMAT (10X,3E17.8)
C          READ(5,1004) IFACC,IFLDS                    FORMAT          (2I5)
C          IF (IFINIT .EQ. 6HINITIL) CALL INTAPE (NWT,TAPEID)
C
C  BLOCK I
C  -------
C
C(1)       READ      (A,NRTC,NCTC,K1,K1)
C(2)       READ      (B,NRTI,NCTI,K1,K1)
C(3)       READ      (A,NRA,NCA,K1,K1)
C(4)       READ      (B,NRPB,NMPB,K1,K1)
C(5)       READIM    (IVEC,NRI,NCI,1,K1)
C          IF (IVEC(1) .NE. 999) GO TO 200
C          DO 210 I = 1,NRPB
C      210 IVEC(I) = I
C(6)   200 READIM    (JVEC,NRJ,NCJ,1,K1)
C          GO TO 110
C(7)   100 READ      (C,NCPB,NCTI,K1,K1)
C(8)       READ      (A,NCTI,NCTI,K1,K1)
C(9)   110 READ      (A,NDPO,NMPO,K1,K1)
C(10)      READIM    (IVEC,NRI,NCI,1,K1)
C(11)      READIM    (JVEC,NRJ,NCJ,1,K1)
C(12)      READ      (ZBR,NRZB,NCZB,1,K1)
C(13)      READ      (OMB,NROB,NCOB,1,K1)
C(14)      READ      (ZSUP,NRZS,NCZS,1,K1)
C(15)      READ      (OMS,NROS,NCOS,1,K1)

C  BLOCK II
C  --------
C(16)      READ      (TSEL,NRS,NCS,K2,K3)
C
C  BLOCK III
C  ---------
C
C          IF (IFACC .EQ. 0) GO TO 500
C(17)      READ (5,1002) (STA(I),I=1,NRP)              FORMAT         (12A6)
C
C  BLOCK IV
C  --------
C
C          IF (IFLDS .EQ. 0) GO TO 500
C(18)      READIM    (LDS,N1,NLDS,1,K1)
           DO 495 L = 1,NLDS
C          NLSS = LDS(L)
C(19)      READ      (C,NMD,NMD,K1,K1)
C          NL = 0
C          DO 300 ILD = 1,NLSS
C          NL = LDS(ILD)
```

```
C(20)        READ      (B,NRB,NCB,K1,K1)
C(21)        READIM    (IVEC,NRI,NCI,1,K1)
C            DO 290 I = 1,NRI
C            IF (IVEC(I) .NE. 0) NL = NL + 1
C            IF (NL .GT. K1) GO TO 300
C        290 CONTINUE
C(22)        READIM    (JVEC,NRJ,NCJ,1,K1)
C        300 CONTINUE
C(23)        READ (5,1002) (STA(I),I=1,NL)                   FORMAT        (12A6)
C        495 CONTINUE
C        500 CONTINUE
C            GO TO 5
```

```
C     ******************************************************************
C
C     INPUT MATRICES
C
C     ******************************************************************
C(1)       A           = COLLAPSE TRANSFORMATION
C(2)       B           = INTERPOLATION TRANSFORMATION, BRANCH RESTRAINT
C                        POINTS TO FORCING FUNCTION COORDINATES
C(3)       A           = CANTILEVERED BRANCH MASS MATRIX
C(4)       B           = BRANCH CANTILEVERED MODES
C(5)       IVEC        = IDENTIFIES ROWS OF PRECEEDING B TO BE KEPT
C(6)       JVEC        = IDENTIFIES COLS OF PRECEEDING B TO BE KEPT
C(7)       C           = TOP PARTITION (NEGATIVE) OF D MATRIX
C(8)       A           = TRIPLE PRODUCT OF MASS MATRIX AND COLLAPSE
C                        TRANSFORMATION
C(9)       A           = SUPPORT FREE-FREE INTERFACE MODES
C(10)      IVEC        = IDENTIFIES ROWS OF PRECEEDING A TO BE KEPT
C(11)      JVEC        = IDENTIFIES COLS OF PRECEEDING A TO BE KEPT
C(12)      ZBR         = BRANCH MODAL DAMPING VECTOR
C(13)      OMB         = BRANCH MODAL FREQUENCY VECTOR (OMEGA SQUARED)
C(14)      ZSUP        = SUPPORT MODAL DAMPING VECTOR
C(15)      OMS         = SUPPORT MODAL FREQUENCY VECTOR (OMEGA SQUARED)
C(16)      TSEL        = FORCE SELECTOR MATRIX
C(17)      STA         = RESPONSE ROW IDENTIFIER
C(18)      LDS         = INTEGER VECTOR OF SIZE NLDS
C                        ELEMENTS = NO. OF SUB LOADS TO COMBINE INTO
C                        ONE LOAD SET
C(19)      C           = MASS MATRIX OF CANTILEVERED BRANCH
C(20)      B           = LOAD COUPLING TRANSFORMATION
C(21)      IVEC        = IDENTIFIES ROWS OF PRECEEDING B TO BE KEPT
C(22)      JVEC        = IDENTIFIES COLS OF PRECEEDING B TO BE KEPT
C(23)      STA         = LOADS ROW IDENTIFIER
```

C. Program Listing

```
      PROGRAM FBBD (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10,
     *          TAPE11,TAPE12,TAPE1,TAPE2,TAPE7)
      DIMENSION A(70,70), B(70,70), C(70,70), D(70,15),
     *           IVEC(70), JVEC(70), ZBR(70), OMB(70), ZSUP(70),
     *           STA(70), TSEL(15,40), VINIT(70), DINIT(70), LDS(70)
      DATA  K1 / K2 / K3 /
     *      70 / 15 / 40 /
      DATA  OMEPS / 1.0E-03 /
C
C   PROGRAM TO GENERATE ACCELERATION, VELOCITY, AND DISPLACEMENT
C   COEFFICIENTS FOR FEED-BACK BASE DRIVE SYSTEM.
C   SOLVE FOR DISCRETE ACCELERATIONS AND INTERNAL LOADS OF BRANCH.
C
C
C
C
C   COUPLED BASE MOTION RESPONSE EQUATION.
C
C
C     ***                  *** **     **    ***                  *** **    **
C     *          -           * *       *    *          -           * *      *
C     * A(1,1) - A(1,2)   * * XDD    *    * B(1,1) -     0     * * XD     *
C     *          -           * *     B *    *          -           * *    B *
C     *-------------------* *-------* + *-------------------* *-------* +
C     *          -           * *       *    *          -           * *      *
C     * A(2,1) - A(2,2)   * * XDD    *    *    0    - B(2,2)   * * XD     *
C     *          -           * *     S *    *          -           * *    S *
C     ***                  *** **     **    ***                  *** **    **
C
C
C        ***                 *** **     **     ***    *** **        **
C        *          -          * *       *     *        * *          *
C        * C(1,1) -     0    * * X     *     * D(1) * * ACC        *
C        *          -          * *     B *  =  *        * *      I/F *
C        *------------------* *-------*     *-------* **          **
C        *          -          * *       *     *        *
C        *     0    - C(2,2) * * X     *     * D(2) *
C        *          -          * *     S *     *        *
C        ***                 *** **     **     ***    ***
C
C
C
C        DEFINE INPUT VARIABLES AND MATRICES
C
C        NCPB    = NO. OF BRANCH MODES TO BE KEPT
C        NRO     = NO. OF SUPPORT I/F DOFS (MAX = 15)
C        NCO     = NO. OF SUPPORT MODES TO BE KEPT
C        NU1     = SCRATCH TAPE UNIT
C        NU2     = SCRATCH TAPE UNIT
C        NRTAPE  = UNIT CONTAINING FORCING FUNCTION TIME HISTORIES
C        NXTAPE  = UNIT ON WHICH OUTPUT HISTORY WILL BE WRITTEN IN
C                  SUBROUTINE TRIFBD
C        NPRT    = PRINT INTERVAL FOR OUTPUT
C        NWT     = FORMA TAPE FOR OUTPUT MATRICES
C        IFMC    = 0, DISCRETE CANTILEVERED BRANCH
C                = 1, MODALLY COUPLED CANTILEVERED BRANCH
C        IFACC   = 1, DISCRETE ACCELERATIONS OF BRANCH WANTED
C                  0, IF ABOVE IS NOT TO BE CALCULATED
C        IFLDS   = 1, INERTIAL LOADS WANTED
C                  0, IF ABOVE IS NOT TO BE CALCULATED
```

```
C      STARTT = INITIAL TIME FOR OUTPUT HISTORY
C      ENDT   = LAST TIME FOR OUTPUT HISTORY
C      DELTAT = FORCING FUNCTION TIME INTERVAL
C
C      ZBR    = BRANCH MODAL DAMPING VECTOR
C      OMB    = BRANCH MODAL FREQUENCY VECTOR (OMEGA SQUARED)
C      ZSUP   = SUPPORT MODAL DAMPING VECTOR
C      OMS    = SUPPORT MODAL FREQUENCY VECTOR (OMEGA SQUARED)
C      TSEL   = FORCE SELLECTOR MATRIX
C      DINIT  = INITIAL DISPLACEMENT (INITIAL ELASTIC DISPLACEMENTS
C                ARE COMPUTED IN SUBROUTINE TR1FBD)
C      VINIT  = INITIAL VELOCITY
C      NLDS   = NO. OF TOTAL LOAD SETS TO CYCLE THRU
C      LDS    = INTEGER VECTOR OF SIZE NLDS
C               ELEMENTS = NO. OF SUB LOADS TO COMBINE
C                           INTO ONE LOAD SET
C
 1001 FORMAT (16I5)
 1002 FORMAT (12A6)
 1003 FORMAT (10X,4E17.8)
 1004 FORMAT (2I5)
C
    5 CALL START
      READ (5,1001) NCPB,NRO,NCO,NU1,NU2,NRTAPE,NXTAPE,NPRT,NWT,IFMC
      READ (5,1002) IFINIT,TAPEID
      READ (5,1003) STARTT,DELTAT,ENDT
      READ (5,1004) IFACC, IFLDS
      IF (IFINIT .EQ. 6HINITIL) CALL INTAPE  (NWT,TAPEID)
C
C
C    BLOCK I - FORMULATION OF COEFFICIENT MATRICES OF
C    -------   ------------------------------------
C              COUPLED BASE MOTION RESPONSE EQUATION.
C              ------------------------------------
C
C
C    READ IN BRANCH MASS MATRIX, COLLAPSE TRANSFORMATION,
C    AND CANTILEVERED MODES.
C
      CALL READ      (A,NRTC,NCTC,K1,K1)
      CALL READ      (B,NRTI,NCTI,K1,K1)
      CALL MULTB     (A,B,NRTC,NRTI,NCTI,K1,K1)
C
      CALL WRITE     (B,NRTC,NCTI,6HT,MASS,K1)
      CALL WTAPE     (B,NRTC,NCTI,6HT,MASS,K1,NWT)
C
      CALL READ      (A,NRA,NCA,K1,K1)
      CALL MULT      (A,B,C,NRA,NCA,NCTI,K1,K1)
      CALL BTABA     (A,B,NRTC,NCTI,K1,K1)
C
      WRITE (NXTAPE) ((A(I,J),I=1,NCTI),J=1,NCTI)
C
      CALL READ      (B,NRPB,NMPB,K1,K1)
      CALL ZERO      (A,NRPB,NCPB,K1)
      CALL READIM    (IVEC,NRI,NCI,1,K1)
      IF (IVEC(1) .NE. 999) GO TO 200
      DO 210 I = 1,NRPB
  210 IVEC(I) = I
  200 CALL READIM    (JVEC,NRJ,NCJ,1,K1)
      CALL REVADD    (1.,B,IVEC,JVEC,A,NRPB,NMPB,NRPB,NCPB,K1,K1)
C
      CALL WRITE     (A,NRPB,NCPB,6HREDCM ,K1)
```

```
      CALL WTAPE     (A,NRPB,NCPB,6HREDCM ,K1,NWT)
C
      CALL TRANS     (A,B,NRPB,NCPB,K1,K1)
      CALL MULTB     (B,C,NCPB,NRA,NCTI,K1,K1)
C
      CALL WRITE     (C,NCPB,NCTI,6H-D(1) ,K1)
C
      GO TO 110
  100 CALL READ      (C,NCPB,NCTI,K1,K1)
      CALL READ      (A,NCTI,NCTI,K1,K1)
      REWIND NXTAPE
C
C
      WRITE (NXTAPE) ((A(I,J),I=1,NCTI),J=1,NCTI)
C
C  READ IN SUPPORT INTERFACE MODES.
C
  110 CALL READ      (A,NDPO,NMPO,K1,K1)
      CALL READIM    (IVEC,NRI,NCI,1,K1)
      CALL READIM    (JVEC,NRJ,NCJ,1,K1)
      CALL ZERO      (B,NRO,NCO,K1)
      CALL REVADD    (1.,A,IVEC,JVEC,B,NDPO,NMPO,NRO,NCO,K1,K1)
C
      CALL WRITE     (B,NRO,NCO,6HREDOM ,K1)
      CALL WTAPE     (B,NRO,NCO,6HREDOM ,K1,NWT)
      REWIND NU1
      WRITE (NU1)  ((C(I,J),I=1,NCPB),J=1,NCTI)
C
      CALL MULTA     (C,B,NCPB,NRO,NCO,K1,K1)
C
      CALL WRITE     (C,NCPB,NCO,6HA(1,2),K1)
      REWIND NU2
      WRITE (NU2)  ((C(I,J),I=1,NCPB),J=1,NCO)
C
      CALL TRANS     (B,C,NRO,NCO,K1,K1)
      REWIND NXTAPE
      READ (NXTAPE)  ((A(I,J),I=1,NCTI),J=1,NCTI)
      REWIND NXTAPE
      CALL MULTB     (C,A,NCO,NCTI,NCTI,K1,K1)
C
      CALL WRITE     (A,NCO,NCTI,6H-D(2) ,K1)
C
      CALL MULTB     (A,B,NCO,NRO,NCO,K1,K1)
      CALL WRITE     (B,NCO,NCO,6HA(2,2),K1)
C
      WRITE (NU1)  ((A(I,J),I=1,NCO),J=1,NCTI)
      WRITE (NU2)  ((B(I,J),I=1,NCO),J=1,NCO)
C
C  READ IN MODAL DAMPING AND FREQUENCY VECTORS, BRANCH AND
C  SUPPORT.
C
      CALL READ      (ZBR,NRZB,NCZB,1,K1)
      CALL READ      (OMB,NROB,NCOB,1,K1)
      CALL READ      (ZSUP,NRZS,NCZS,1,K1)
      CALL READ      (OMS,NROS,NCOS,1,K1)
C
C  FORM A, B, C AND D MATRICES FOR EQ. OF MOTION.
C
      NMD = NCPB + NCO
      CALL ZERO      (A,NMD,NMD,K1)
      CALL ZERO      (B,NMD,NMD,K1)
      CALL ZERO      (C,NMD,NMD,K1)
```

```
      CALL ZERO      (D,NMD,NCTI,K1)
      DO 10 I = 1,NCPB
      A(I,I) = 1.0
      IF (OMB(I) .LE. OMEPS) OMB(I) = 0.0
      B(I,I) = 2.0 * ZBR(I) * SQRT(OMB(I))
      C(I,I) = OMB(I)
   10 CONTINUE
C
C  USE MEQ. FOR SUPPORT = 1.0
C
      DO 20 I = 1,NCO
      IF (OMS(I) .LE. OMEPS) OMS(I) = 0.0
      B(I+NCPB,I+NCPB) = 2.0 * ZSUP(I) * SQRT(OMS(I))
      C(I+NCPB,I+NCPB) = OMS(I)
   20 CONTINUE
C
      REWIND NU1
      READ (NU1)  (D(I,J),I=1,NCPB),J=1,NCTI)
      READ (NU1) ((D(I+NCPB,J)I=1,NCO),J=1,NCTI)
      DO 30 I = 1, NMD
      DO 40 J = 1,NCTI
      D(I,J) = -D(I,J)
   40 CONTINUE
   30 CONTINUE
      REWIND NU2
      READ (NU2) ((A(I,J+NCPB),I=I,NCPB),J=I,NCO)
      DO 50 I = 1,NCPB
      DO 60 J = 1,NCO
      A(J+NCPB,I) = A(I,J+NCPB)
   60 CONTINUE
   50 CONTINUE
      READ (NU2) ((A(I+NCPB,J+NCPB),I=1,NCO),J=1,NCO)
      DO 70 I = 1, NCO
      A(I+NCPB,I+NCPB) = 1.0 + A(I+NCPB,I+NCPB)
   70 CONTINUE
C
      CALL WRITE     (A,NMD,NMD,6HCOEF,A,K1)
      CALL WTAPE     (A,NMD,NMD,6HCOEF,A,K1,NWT)
      CALL WRITE     (B,NMD,NMD,6HCOEF,B,K1)
      CALL WRITE     (C,NMD,NMD,6HCOEF,C,K1)
      CALL WTAPE     (C,NMD,NMD,6HCOEF,C,K1,NWT)
      CALL WRITE     (D,NMD,NCTI,6HCOEF,D,K1)
C
C
C  BLOCK II - SOLUTION OF COUPLED BASE MOTION RESPONSE EQUATION.
C  --------   -----------------------------------------------
C
C
C  READ SELECTOR MATRIX FOR DRIVING ACCELERATIONS.
C  ZERO INITIL DISPLACEMENTS, READ IN INITIL VELOCITIES.
C
      CALL READ      (TSEL,NRS,NCS,K2,K3)
      CALL ZERO      (DINIT,1,NMD,1)
      CALL ZERO      (VINIT,1,NMD,1)
C
C  GENERATE MODAL ACCELERATIONS, VELOCITIES, AND DISPLACEMENTS.
C
      REWIND NWT
      CALL TRIFBD    (A,B,C,D,TSEL,VINIT,DINIT,STARTT,DELTAT,ENDT,
     *                NPRT,NMD,NRS,NCS,K1,K2,K3,NXTAPE,NRTAPE)
C
C
```

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

```
C  BLOCK III - CALCULATION OF DISCRETE ACCELERATIONS OF BRANCH.
C  ---------   ------------------------------------------------
C
C
      IF (IFACC .EQ. 0) GO TO 500
      REWIND NWT
      CALL ZERO      (A,K1,K1,K1)
      CALL RTAPE     (IARUNO,6HREDCM ,A,NRP,NCP,K1,K1,NWT)
      CALL RTAPE     (IARUNO,6HREDOM ,B,NRO,NCO,K1,K1,NWT)
      CALL RTAPE     (IARUNO,6HT,MASS,C,NRR,NCR,K1,K1,NWT)
      READ (5,1002)  (STA(I),I=1,NRP)
      CALL TACCBD    (A,B,C,STA,1.0,NRP,NRS,NMD,NXTAPE,NU1,
     *                NPRT, STARTT,ENDT,K1)
C
C
C  BLOCK IV - CALCULATION OF INTERNAL LOADS.
C---------   ----------------------------
C
C
      IF (IFLDS .EQ. 0) GO TO 500
      CALL READIM    (LDS,N1,NLDS,1,K1)
      DO 495 L = 1,NLDS
      NLSS = LDS(L)
      CALL READ      (C,NMD,NMD,K1,K1)
      CALL ZERO      (A,K1,K1,K1)
      NL = 0
      DO 300 ILD = 1, NLSS
      NL = LDS(ILD)
      CALL READ      (B,NRB,NCB,K1,K1)
      CALL READIM    (IVEC,NRI,NCI,1,K1)
      DO 290 I = 1,NRI
      IF (IVEC(I) .NE. 0) NL = NL + 1
      IF (NL .GT. K1) GO TO 300
  290 CONTINUE
      CALL READIM    (JVEC,NRJ,NCJ,1,K1)
      CALL REVADD    (1.0,B,IVEC,JVEC,A,NRB,NCB,NL,NMD,K1,K1)
  300 CONTINUE
      CALL MULTA     (A,C,NL,NMD,NMD,K1,K1)
      DO 400 I = 1,NL
      DO 400 J = 1,NMD
      A(I,J) = -A(I,J)
  400 CONTINUF
      REWIND NU1
      READ (5,1002)  (STA(I),I=1,NL)
      CALL ACCLD     (A,STA,STARTT,ENDT,NL,NMD,NPRT,K1,NU1)
  495 CONTINUE
  500 CONTINUE
      CALL LTAPE     (NWT)
      GO TO 5
      END
```

```
      SUBROUTINE TRIFBD (A,B,C,D,TSEL,XD0,X0,STARTT,DELTAT,ENDT,
     1                 NWRITE,NX,NF,NCS,KA,KF,KT,NTAPE,NRTAPE)
      DIMENSION A(KA,1),B(KA,1),C(KA,1),D(KA,1),TSEL(KF,1),
     *          AC(42),F1(15),F2(15),F(15),QD(130),Q(130),P(4),
     *          FC(70),
     *          XD0(1),X0(1),XDD(130),XD(130),X(130),
     *          XDDMAX(130),  XDMAX(130),  XMAX(130),
     *          XDDMIN(130),  XDMIN(130),  XMIN(130),
     *          TDDMAX(130), TXDMAX(130), TXMAX(130),
     *          TDDMIN(130), TXDMIN(130), TXMIN(130)
      DATA NIT,NOT/5,6/
      DATA NLPP / 60 /
C
C  RESPONSE ROUTINE TO SOLVE THE SECOND ORDER DIFFERENTIAL EQUATION
C      (A)XDD + (B)XD + (C)X = (D)F  FOR XDD, XD, X.
C  FOURTH ORDER RUNGE-KUTTA (GILL MODIFICATION) NUMERICAL INTEGRATION
C  IS USED.
C  MATRICES A,B,C,D SHOULD NOT SHARE SAME CORE LOCATION (DUE TO MULTB).
C  THE ANSWERS (T,F,XDD,XD,X) WILL BE WRITTEN ON NTAPE EVERY DELTAT AND
C  ON PAPER EVERY NWRITE * DELTAT.
C  CALLS FORMA SUBROUTINES INV1,MULT,MULTB,PAGEHD,ZZBOMB.
C  THE MAXIMUM SIZES ARE (BASED ON DIMENSIONS OF XDD,XD,X,F,MAX,MINS)
C      NX = 130
C      NF = 15
C
C      SUBROUTINE ARGUMENTS (ALL INPUT)
C  A      = MATRIX COEFFICIENT OF XDD. SIZE (NX,NX). * DESTROYED *
C  B      = MATRIX COEFFICIENT OF XD.  SIZE (NX,NX). * DESTROYED *
C  C      = MATRIX COEFFICIENT OF X.   SIZE (NX,NX). * DESTROYED *
C  D      = MATRIX COEFFICIENT OF F.   SIZE (NX,NF). * DESTROYED *
C  TSEL   = FORCE SELECTOR MATRIX. SIZE (NF,NCS).
C  XD0    = VECTOR OF INITIAL VELOCITIES. SIZE (NX).
C  X0     = VECTOR OF INITIAL DISPLACEMENTS. SIZE (NX).
C  STARTT= START TIME.
C  DELTAT= INTEGRATION STEP SIZE.
C  ENDT   = END TIME.
C  NWRITE= MULTIPLE OF INTEGRATION STEPS TO WRITE ON PAPER.
C          NWRITE = 1   WRITE EVERY STEP (0,1,2,...)
C          NWRITE = 2   WRITE EVERY SECOND STEP (0,2,4,...)
C                ETC
C  NX     = SIZE OF MATRICES A,B,C (SQUARE). NUMBER OF ROWS IN D
C  NF     = NUMBER OF COLS IN D. MAX=15
C  NCS    = NUMBER OF COLS IN TSEL. MAX=40.
C  KA     = ROW DIMENSION OF A,B,C,D IN CALLING PROGRAM.
C  KF     = ROW DIMENSION OF TSEL IN CALLING PROGRAM.
C  KT     = COL DIMENSION OF TSEL IN CALLING PROGRAM.
C  NTAPE = NUMBER OF TAPE ON WHICH ANSWERS WILL BE WRITTEN. (E.G. 10).
C  NRTAPE= NUMBER OF TAPE CONTAINING DRIVING FORCES. (E.G. 11).
C
C      THE OUTPUT DATA (TO BE WRITTEN ON PAPER AND TAPE) IS
C  T      = TIME
C  F      = FORCE OBTAINED BY LINEAR INTERPOLATION ON TABF. SIZE (NF).
C  XDD    = ACCELERATION. SIZE (NX).
C  XD     = VELOCITY.     SIZE (NX).
C  X      = DISPLACEMENT. SIZE (NX).
C
 2001 FORMAT (////15X,40H THE INPUT SCALARS TO SUBROUTINE TRI ARE ,
     1            //23X,          10H STARTT = F10.6,
     2            //23X,          10H DELTAT = F10.6,
```

```
     3              //23X,          10H ENDT   = F10.6,
     4              //23X,          10H NWRITE = I5     )
 2010 FORMAT (// 20X,8H TIME OF, 28X,8H TIME OF
     1          /9X,4H ROW, 6X,10H MAX ACCEL, 6X,10H MAX ACCEL,
     2           10X,10H MIN ACCEL, 6X,10H MIN ACCEL
     3           // (10X, I3, F16.6, E18.8, F18.6, E18.8))
 2020 FORMAT (// 20X,8H TIME OF, 28X,8H TIME OF
     1          /9X,4H ROW, 7X,8H MAX VEL, 8X,8H MAX VEL,
     2           12X,8H MIN VEL, 8X,8H MIN VEL
     3           // (10X, I3, F16.6, E18.8, F18.6, E18.8))
 2030 FORMAT (// 20X,9H TIME OF, 28X,8H TIME OF
     1          /9X,4H ROW, 7X,9H MAX DISP, 7X,9H MAX DISP,
     2           10X,9H MIN DISP, 8X,9H MIN DISP
     3           // (10X, I3, F16.6, E18.8, F18.6, E18.8))
 2040 FORMAT (//9X,8H TIME = F10.6)
 2050 FORMAT (//9X,15H APPLIED FORCES / (10X, 5E16.8))
 2060 FORMAT (// 9X,4H ROW, 6X,13H ACCELERATION, 8X,9H VELOCITY,
     1           10X,13H DISPLACEMENT // (10X, I3, 3E20.8))
C
                                                              NERROR=1
      IF (NX .GT. 130 .OR. NF .GT. 15) GO TO 999
C
C  COMPUTE INITIL DISPLACEMENTS.
C
      REWIND NTAPE
      WRITE (NTAPE) ((A(I,J),I=1,NX),J=1,NX)
      WRITE (NTAPE) ((C(I,J),I=1,NX),J=1,NX)
      REWIND NTAPE
      REWIND NRTAPE
      READ (NRTAPE) T, (FC(I),I=1,NCS)
C
C  FC = INTERFACE ACCELERATIONS AT T = STARTT.
      CALL WRITE (FC,1,NCS,2HFC,1)
C
      CALL MULTB (TSEL,FC,NF,NCS,1,KF,KA)
      CALL MULTB (D,FC,NX,NF,1,KA,KA)
      DO 70 J = 1, NX
      IF (C(J,J) .NE. 0.) GO TO 70
      DO 65 I = 1, NX
      C(I,J) = A(I,J)
   65 CONTINUE
   70 CONTINUE
      CALL INVI (C,A,NX,KA)
      CALL MULT (A,FC,X0,NX,NX,1,KA,KA)
      READ (NTAPE) ((A(I,J),I=1,NX),J=1,NX)
      READ (NTAPE) ((C(I,J),I=1,NX),J=1,NX)
      DO 80 I = 1, NX
      ACC(I) = ACC(I) + RBMD(I,J) * F(J)
      IF (C(I,I) .EQ. 0.) X0(I)=0.0
   80 CONTINUE
      REWIND NRTAPE
C
C  PRINT INPUT SCALARS.
C
      CALL PAGEHD
      WRITE (NOT,2001) STARTT,DELTAT,ENDT,NWRITE
C
      REWIND NTAPE
      WRITE (NTAPE) ((B(I,J), I=1,NX), J=1,NX)                    B=B
      CALL INVI   (A, B, NX, KA)                                   B=AI
      WRITE (NTAPE) ((B(I,J), I=1,NX), J=1,NX)
      REWIND NTAPE
```

```
      READ  (NTAPE) ((B(I,J), I=1,NX), J=1,NX)                         B=B
      READ  (NTAPE) ((A(I,J), I=1,NX), J=1,NX)                         A=AI
      CALL MULTB  (A, B, NX, NX, NX, KA, KA)                           B=AIB
      CALL MULTB  (A, C, NX, NX, NX, KA, KA)                           C=AIC
      CALL MULTB  (A, D, NX, NX, NF, KA, KA)                           D=AID
      REWIND NTAPE
      CALL WRITE (B,NX,NX,6HINVA*B,KA)
      CALL WRITE (C,NX,NX,6HINVA*C,KA)
      CALL WRITE (D,NX,NF,6HINVA*D,KA)
C
C  SET INITIAL VALUES.
C
      NSTEPS = 0
      T = STARTT
      DO 30 I=1,NX
      QD(I) = 0.0
      Q (I) = 0.0
      XD(I) = XD0(I)
   30 X (I) = X0 (I)
      REWIND NRTAPE
      READ (NRTAPE) T1, (AC(I),I=1,NCS)
      CALL WRITE (AC,1,NCS,2HAC,1)
      CALL MULT      (ISEL,AC,F1,NF,NCS,1,KF,KT)
      CALL WRITE (F1,1,NF,2HF1,1)
      DO 205 I = 1,NF
  205 F(I) = F1(I)
      DO 38 I=1,NX
      XDD(I) = 0.0
      DO 37 J=1,NF
   37 XDD(I) = XDD(I) +   D(I,J)*F(J)
      DO 38 J=1,NX
   38 XDD(I) = XDD(I) -   B(I,J)*XD(J) -   C(I,J)*X(J)
      DO 40 I=1,NX
      XDDMAX(I) = XDD(I)
      XDDMIN(I) = XDD(I)
      TDDMAX(I) = STARTT
      TDDMIN(I) = STARTT
       XDMAX(I) = XD(I)
       XDMIN(I) = XD(I)
      TXDMAX(I) = STARTT
      TXDMIN(I) = STARTT
       XMAX(I)  = X(I)
       XMIN(I)  = X(I)
      TXMAX(I)  = STARTT
   40 TXMIN(I)  = STARTT
C
C  SET INTEGRATION CONSTANTS.
C
      P(1) = .5
      P(2) = 1. - SQRT(.5)
      P(3) = 1. + SQRT(.5)
      P(4) = .5
C
C  WRITE DATA AT START.
C
      NW = NWRITE
      GO TO 340
C
C  INTEGRATION LOOP. (J=1,HALF STEP), (J=2,HALF STEP AGAIN),
C                    (J=3,FULL STEP), (J=4,END OF STEP).
C  GILL FACTOR = .5
C
```

```
100 READ (NRTAPE) T1, (AC(I),I=1,NCS)
    CALL MULT      (ISEL,AC,F2,NF,NCS,1,KF,KT)
    DO 150 J=1,4
    DO 110 I=1,NX
    Z  = XD (I) * DELTAT
    ZD = XDD(I) * DELTAT
    IF (J .EQ. 4) GO TO 105
    R  = P(J) * (Z -Q (I))
    RD = P(J) * (ZD-QD(I))
    GO TO 107
105 R  = (Z  - 2.*Q (I))/6.
    RD = (ZD - 2.*QD(I))/6.
107 X (I) = X (I) + R
    XD(I) = XD(I) + RD
    Q (I) = Q (I) + 3.*R  - P(J)*Z
110 QD(I) = QD(I) + 3.*RD - P(J)*ZD
    GO TO (200,140,220,140),J
200 DO 210 K = 1,NF
210 F(K) = (F1(K) + F2(K)) / 2.0
    GO TO 140
220 DO 230 K = 1,NF
230 F(K) = F2(K)
    NSTEPS = NSTEPS + 1
    T = STARTT + FLOAT(NSTEPS)*DELTAT
140 DO 150 I=1,NX
    XDD(I) = 0.0
    DO 145 K=1,NF
145 XDD(I) = XDD(I) +   D(I,K)*F(K)
    DO 150 K=1,NX
150 XDD(I) = XDD(I) -   B(I,K)*XD(K) -   C(I,K)*X(K)
    DO 240 I = 1,NF
240 F1(I) = F2(I)
C
C  GET MAXIMUMS AND MINIMUMS.
C
    DO 330 I=1,NX
    IF (XDD(I) .LE. XDDMAX(I)) GO TO 305
    XDDMAX(I) = XDD(I)
    TDDMAX(I) = T
305 IF (XDD(I) .GE. XDDMIN(I)) GO TO 310
    XDDMIN(I) = XDD(I)
    TDDMIN(I) = T
310 IF (XD(I) .LE. XDMAX(I)) GO TO 315
     XDMAX(I) = XD(I)
    TXDMAX(I) = T
315 IF (XD(I) .GE. XDMIN(I)) GO TO 320
     XDMIN(I) = XD(I)
    TXDMIN(I) = T
320 IF (X(I) .LE. XMAX(I)) GO TO 325
     XMAX(I) = X(I)
    TXMAX(I) = T
325 IF (X(I) .GE. XMIN(I)) GO TO 330
     XMIN(I) = X(I)
    TXMIN(I) = T
330 CONTINUE
C
C  WRITE ANSWERS ON NTAPE FOR SUBSEQUENT USE (SUCH AS
C  TIME RESPONSE ADDITIONAL EQUATIONS OR PLOT).
C
340 WRITE (NTAPE) T, (F(J), J=1,NF), (XDD(I),XD(I),X(I), I=1,NX)
C
C  SEE IF DATA SHOULD BE PRINTED.
```

```
C
      IF (NW .LT. NWRITE) GO TO 345
      CALL PAGEHD
      WRITE (NOT,2040) T
      WRITE (NOT,2050) (F(I), I=1,NF)
      NXS = 1
      NXE = NX
      NFLN = (NF-1)/5+1
      IF ((NXE + NFLN) .GT. (NLPP-15)) NXE=(NLPP-15)-NFLN
  342 WRITE (NOT,2060) (I, XDD(I), XD(I), X(I), I=NXS,NXE)
      IF (NX .EQ. NXE)  GO TO 343
      NXS = NXE + 1
      NXE = NX
      IF ((NXE-NXS) .GT. (NLPP- 9)) NXE=NXS+(NLPP- 9)
      CALL PAGEHD
      GO TO 342
  343 NW = 0
  345 NW = NW+1
C
C   SEE IF RUN HAS DIVERGED.
C
                                                                 NERROR=2
      DO 350 I=1,NX
      IF (ABS(X(I)) .GT. 1.E+35) GO TO 999
  350 CONTINUE
C
C   DETEPMINE IF RUN IS FINISHED.
C
      IF (T .LT. ENDT) GO TO 100
C
C   PRINT MAXIMUMS AND MINIMUMS IF RUN IS FINISHED.
C
      DO 410 MM=1,3
      NXE = 0
  400 NXS = NXE + 1
      NXE = NX
      IF ((NXE-NXS) .GT. (NLPP-11)) NXE=NXS+(NLPP-11)
      CALL PAGEHD
      IF (MM .EQ. 1) WRITE (NOT,2010) (I,TDDMAX(I),XDDMAX(I),
     *                                  TDDMIN(I),XDDMIN(I), I=NXS,NXE)
      IF (MM .EQ. 2) WRITE (NOT,2020) (I,TXDMAX(I),XDMAX(I),
     *                                  TXDMIN(I),XDMIN(I), I=NXS,NXE)
      IF (MM .EQ. 3) WRITE (NOT,2030) (I,TXMAX(I),XMAX(I),
     *                                  TXMIN(I),XMIN(I), I=NXS,NXE)
      IF (NX .GT. NXE) GO TO 400
  410 CONTINUE
      RETURN
C
  999 CALL ZZBOMB (6HTRIFBD,NERROR)
      END
```

```
      SUBROUTINE TACCBD (PHIBR,PSUP,RBMD,STA,CONVT,NRP,NF,NX,
     *                   NRTAPE,NXTAPE,NPRINT,STARTT,ENDT,KR)
      DIMENSION PHIBR(KR,1), PSUP(KR,1), RBMD(KR,1), STA(1),
     *          F(15), XDD(70), ACC(70), ACCMAX(70,4), TIME(5),
     *          THIST(70,5)
C
C
C  PROGRAM TO GET TOTAL ACCELERATIONS FOR BASE-DRIVEN PAYLOAD
C         TOTACC = PHI(CANT) * BDACC
C                   + PHI(RB) * PHI(SUPPORT SYS) * SUPPORT ACC
C                   + PHI(RB) * BASE ACC
C
C  DEFINITION OF INPUT VARIABLES
C
C     NRP    = NUMBER OF RESPONSES TO BE CALCULATED
C                  (NUMBER OF ROWS IN PHIBR)
C     NF     = NUMBER OF PSEUDO-FORCES USED TO DRIVE MODEL BASE
C     NX     = NUMBER OF TOTAL MODES CONSIDERED IN THE RESPONSE EQUATION
C              OF THE COUPLED SYSTEM
C     NRTAPE = LOGICAL UNIT OF DRIVING RESPONSE TAPE
C     NXTAPE = LOGICAL UNIT OF OUTPUT RESPONSE TAPE
C     NPRINT = PRINT INTERVAL FOR RESPONSE CALCULATIONS
C     STARTT = START TIME FOR RESPONSE CALCULATIONS
C     ENDT   = END TIME FOR RESPONSE CALCULATIONS
C      ONVT  = SCALE FACTOR FOR ANSWERS
C     PHIBR  = CANTILEVERED MODES FOR DRIVEN SYSTEM
C     PSUP   = REDUCED FREE-FREE MODES OF DRIVING SYSTEM
C     RBMD   = REDUCTION TRANSFORMATION FOR DRIVEN SYSTEM,
C              RELATIVE TO BASE
C     STA    = RESPONSE ROW IDENTIFIER
C
C
C
 1002 FORMAT (10X,4E17.8)
 1003 FORMAT (12A6)
 2001 FORMAT (//56X,*ACCELERATION TIME HISTORY*//10X,*STA   *,
     *          5(7X,*TIME = *,F8.5)/)
 2002 FORMAT (2X,I5,2X,A6,5(7X,E15.8))
C
      CALL MULT     (RBMD,PSUP,PHIBR(1,NCP+1),NRR,NRO,NCO,KR,KR)
      CALL WRITE    (PHIBR,NRP,NX,6HC,QDD ,KR)
      REWIND NRTAPE
      REWIND NXTAPE
C
C  READ PREVIOUSLY WRITTEN TAPE AND CALCULATE ACCELERATIONS
C
      NTIME = 0
      IPRINT = 0
  110 READ (NRTAPE) T,(F(J),J=1,NF),(XDD(I),XD,X,I=1,NX)
      CALL ZERO     (ACC,NRP,1,KR)
      DO 20 I = 1,NRP
      DO 30 J = 1,NF
      ACC(I) = ACC(I) + RBMD(I,J) * F(J)
   30 CONTINUE
      DO 40 J = 1,NX
      ACC(I) = ACC(I) + PHIBR(I,J) * XDD(J)
   40 CONTINUE
      ACC(I) = ACC(I) * CONVT
   20 CONTINUE
```

```
      IF (NXTAPE .NE. 0) WRITE (NXTAPE) T,(ACC(J),J=1,NRP)
C
C  SEE IF MAX OR MIN HAS BEEN CALCULATED AT THIS TIME
C
      IF (T .GT. STARTT) GO TO 50
      CALL INTMAX    (ACC,ACCMAX,NRP,T,KR)
      GO TO 60
   50 CALL MAX1      (ACC,ACCMAX,NRP,T,KR)
   60 CONTINUE
C
C  PRINT DATA
C
      IPRINT = IPRINT + 1
      IF (IPRINT .EQ. NPRINT .OR. T .EQ. STARTT .OR. T .EQ. ENDT)
     *         GO TO 70
      GO TO 80
C
   70 NTIME = NTIME + 1
      IPRINT = 0
      TIME(NTIME) = T
      CALL ASSEM    (ACC,1,NTIME,THIST,NRP,1,NRP,5,K1,K1)
      IF (NTIME .EQ. 5 .OR. T .EQ. ENDT) GO TO 90
      GO TO 80
C
   90 CALL PAGEHD
      WRITE (6,2001) (TIME(J),J=1,NTIME)
      NLINE = 0
      DO 100 I = 1,NRP
      NLINE = NLINE + 1
      WRITE (6,2002) I,STA(I),(THIST(I,J),J=1,NTIME)
      IF (NLINE .LT. 40) GO TO 100
C
      CALL PAGEHD
      WRITE (6,2001) (TIME(J),J=1,NTIME)
      NLINE = 0
  100 CONTINUE
      NTIME = 0
   80 IF (T .LT. ENDT) GO TO 110
      CALL WRITE    (ACCMAX,NRP,4,6HACCMAX,KR)
      RETURN
      END
```

```
      SUBROUTINE INTMAX (A,B,NRA,VAR,KRA)
      DIMENSION A(1),B(KRA,1)
C
C  THIS PROGRAM INITIALIZES MATRIX B WITH THE VALUES IN VECTOR A AND
C  THE VARIABLE VAR
C
C  INPUT VARIABLES
C
C     A     = VECTOR OF VALUES TO BE STORED IN COLUMNS 2 AND 4 OF B
C     B     = OUTPUT RESULTANT MATRIX
C     NRA   = NUMBER OF ELEMENTS IN A
C     VAR   = VARIABLE TO BE STORED IN COLUMNS 1 AND 3 OF B
C     KRA   = DIMENSION SIZE OF A AND ROW SIZE OF B IN CALLING PROGRAM
C
      DO 100 IRA=1,NRA
      B(IRA,1)=VAR
      B(IRA,3)=VAR
      B(IRA,2)=A(IRA)
      B(IRA,4)=A(IRA)
  100 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MAX1 (A,B,NRA,VAR,KRA)
      DIMENSION A(1),B(KRA,1)
C
C  THIS SUBROUTINE COMPARES THE VALUES IN VECTOR A WITH THE VALUES
C  IN COLUMNS 2 AND 4 OF MATRIX B
C     IF(A(I).GT.B(I,2)) B(I,1)=VAR
C                        B(I,2)=A(I)
C     IF(A(I).LT.B(I,4)) B(I,3)=VAR
C                        B(I,4)=A(I)
C
C  INPUT VARIABLES
C
C     A      = VECTOR CONTAINING VALUES TO BE COMPARED WITH VALUES
C              IN B
C     B      = (NRA,4) MATRIX OF MAX AND MINS
C     NRA    = NUMBER OF ELEMENTS IN A
C     VAR    = A CONSTANT WHICH REPLACES VALUES IN COLUMNS 1 AND 3 OF B
C     KRA    = DIMENSION SIZE OF A AND ROW SIZE OF B IN CALLING PROGRAM
C
      DO 110 IRA=1,NRA
      IF(B(IRA,2).GT.A(IRA)) GO TO 100
      B(IRA,1)=VAR
      B(IRA,2)=A(IRA)
  100 IF(B(IRA,4).LT.A(IRA)) GO TO 110
      B(IRA,3)=VAR
      B(IRA,4)=A(IRA)
  110 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ACCLD (A,STA,STARTT,ENDT,NL,NX,NPRINT,KA,NRTAPE)
      DIMENSION A(KA,1),STA(1), ZLD(70),ZLDMAX(70,4),
     *          TIME(5), THIST(70,5)
      DATA KZ / 70 /
C
C   THIS SUBROUTINE CALCULATES INTERNAL LOADS IN BRANCH
C
C
C   INPUT VARIABLES
C
C     A        = LOADS TRANSFORMATION (DISCRETE ACCELERATION TYPE)
      STA      = LOAD ROW IDENTIFIER
C     STARTT   = START TIME OF RESPONSE CALCULATED FOR BRANCH
C     ENDT     = END TIME OF RESPONSE CALCULATED FOR BRANCH
C     NL       = NUMBER OF ROWS OF MATRIX A
C     NX       = NUMBER OF COLS OF MATRIX A
C     KA       = ROW DIMENSION OF A IN CALLING PROGRAM
C     NRTAPE   = NUMBER OF TAPE CONTAINING DISCRETE ACCELERATION
C                DATA OF BRANCH
C
      NTIM = 0
      IPRINT = 0
  110 READ (NRTAPE) T,(ZLD(J),J=1,NX)
      CALL MULTB    (A,ZLD,NL,NX,1,KA,KZ)
C
C   SEE IF MAX. OR MIN. HAS BEE CALCULATED AT THIS TIME
C
      IF (T .GT. STARTT) GO TO 50
      CALL INTMAX   (ZLD,ZLDMAX,NL,T,KZ)
      GO TO 60
   50 CALL MAX1     (ZLD,ZLDMAX,NL,T,KZ)
   60 CONTINUE
C
C   PRINT DATA
C
      IPRINT = IPRINT + 1
      IF (IPRINT .EQ. NPRINT .OR. T .EQ. STARTT .OR. T .EQ. ENDT)
     *       GO TO 70
      GO TO 80
C
   70 NTIME = NTIME + 1
      IPRINT = 0
      TIME(NTIME) = T
      CALL ASSEM    (ZLD,1,NTIME,THIST,NL,1,NL,5,KZ,KZ)
      IF (NTIME .EQ. 5 .OR. T .EQ. ENDT) GO TO 90
      GO TO 80
C
   90 CALL PAGEHD
      WRITE (6,2001) (TIME(J),J=1,NTIME)
      NLINE = 0
      DO 100 I = 1,NL
      NLINE = NLINE + 1
      WRITE (6,2002) I,STA(I),(THIST(I,J),J=1,NTIME)
      IF (NLINE .LT. 40) GO TO 100
C
      CALL PAGEHD
      WRITE (6,2001) (TIME(J),J=1,NTIME)
      NLINE = 0
  100 CONTINUE
```

```
80 IF (T .LT. ENDT) GO TO 110
   CALL WRITE    (ZLDMAX,NL,4,6HZLDMAX,KZ)
   RETURN
   END
```

APPENDIX A

Salient Mathematical Features of the Coupled Base Motion Response Technique

Salient Mathematical Features of the Proposed Method:

Schematically the basic method to determine the dynamic response of any branch system using the response analysis of the interface, obtained from the response analysis of an integrated system (base-branch) incorporating a different (or no) branch, can be depicted as shown in Fig. 1A.

For mathematical convenience, the derivations in part (b) of Fig. 1A, namely the coupling of an alternative branch to form a new integrated system, will be presented first, followed by part (a).

The transient response equation for the branch subsystem denoted herein as component B, subject to interface motion excitation only, can be written as the following two dynamical subsystem equations (reference 5):

$$[M]_B\{\ddot{x}_C\}_B + [C]_B\{\dot{x}_C\}_B + [K]_B\{x_C\}_B = -[M]_B[\beta]\{\ddot{\delta}(t)\} \tag{1}$$

and $$\{x\}_B = [\beta]\{\delta(t)\} + \{x_C\}_B \tag{2}$$

where $\{x\}_B$ = Displacement of branch B in complete system

$\{x_C\}_B$ = Displacement of branch B with constrained interface displacement

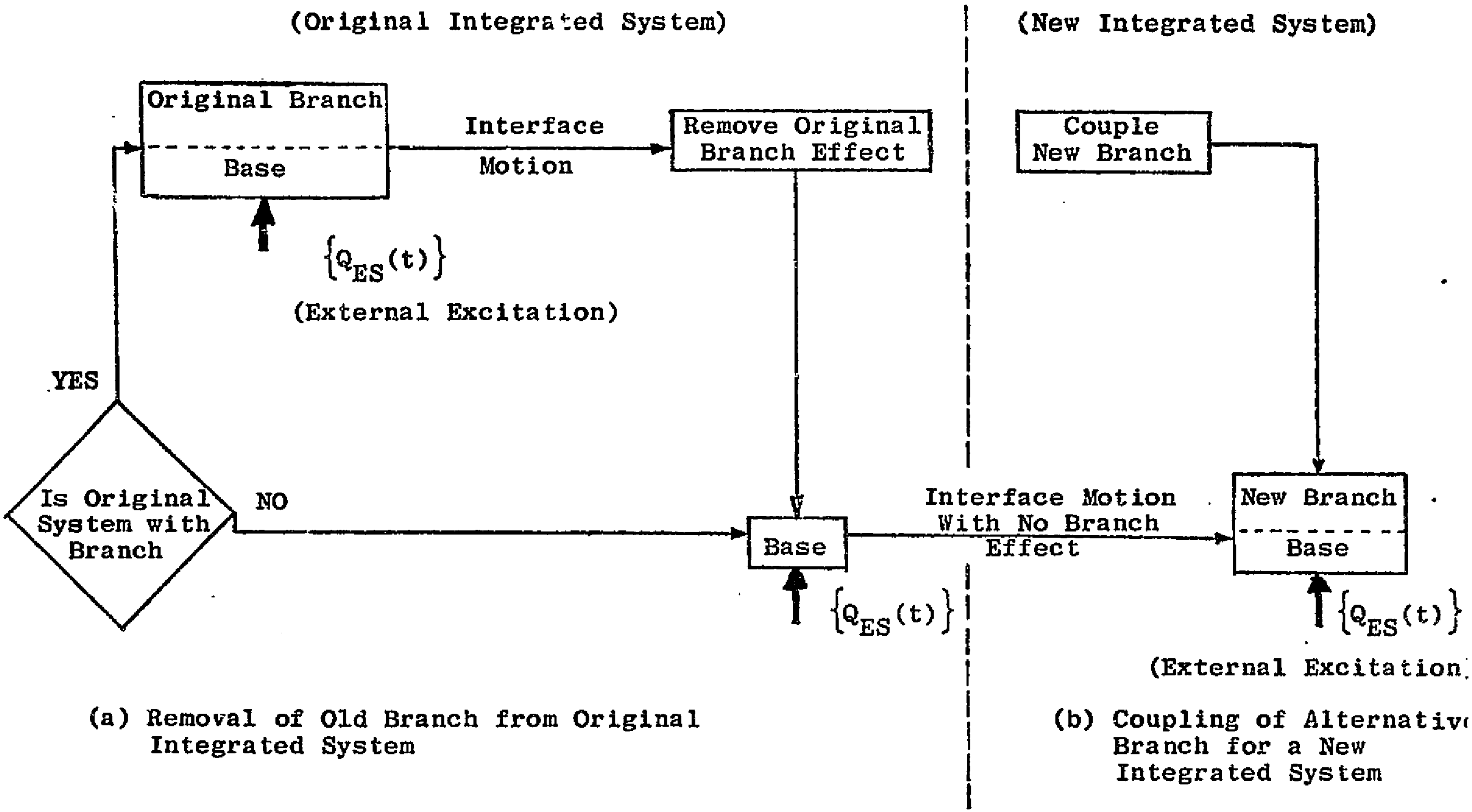


Fig. 1A

Schematic Diagram of the Proposed Method

$\{\delta(t)\}$ = Interface displacements of the support system

$[\beta]$ = Collapse transformation

$[M]_B$ = Mass matrix of constrained branch

$[C]_B$ = Partitioned damping matrix

$[K]_B$ = Partitioned stiffness matrix

Physically, Eq. (1) represents the constrained branch, and Eq. (2) represents the free branch of the dynamical subsystem. Applying the transformation equation

$$\{x_C\}_B = [\varphi_C]_B \{q\}_B \tag{3}$$

where $\{q\}_B$ = Vector of normal modal coordinates of constrained branch

$[\varphi_C]_B$ = Cantilevered mode shapes, normalized to yield unity generalized masses

and substituting Eq. (3) into Eqs. (1) and (2) yields

$$\{\ddot{q}\}_B + \left[\diagdown 2\rho\omega \diagdown\right]_B \{\dot{q}\}_B + \left[\diagdown \omega^2 \diagdown\right]_B \{q\}_B = - [\varphi_C]_B^T [M]_B [\beta] \{\ddot{\delta}(t)\} \tag{4}$$

and $$\{x\}_B = [\beta]\{\delta(t)\} + [\varphi_C]_B \{q\}_B \tag{5}$$

where ω = circular frequency

ρ = Ratio of viscous damping and critical viscous damping (i.e. c/c_R).

Equations (4) and (5) would be directly applicable to the problem only when the interface motion $\{\ddot{\delta}(t)\}$ was derived from an integrated system analysis incorporating a branch model identical to that used in Eq. (4).

The new branch (or component B) when coupled to the base or support (referred to as component S) will result in new interface forces differing from those resulting from the original branch load-class. The interface forces are used to couple the new branch to the base system in the interface motion response equation. Finally the situation as shown in Fig.2A is obtained.

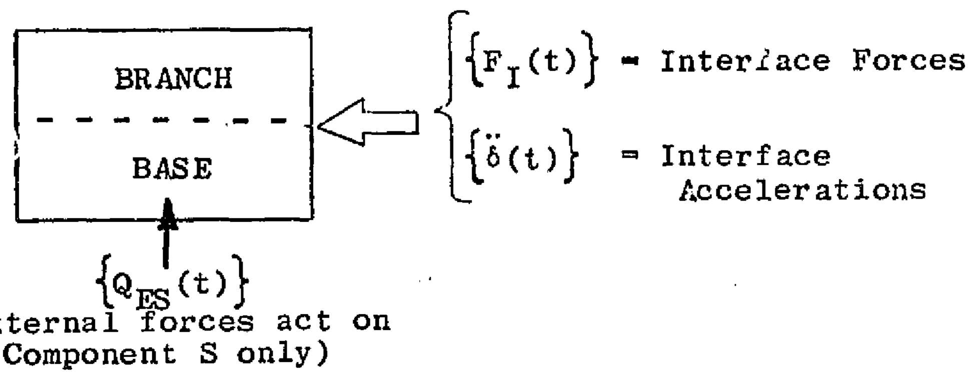


Fig. 2A

The base motion accelerations input to part (b) of Fig. 1A correspond to the acceleration of the interface coordinates of component S resulting from the external forcing function $\{Q_{ES}(t)\}$ only; consequently we denote this interface motion as $\{\delta_{ES}(t)\}$.

Similarly, we denote the acceleration of the interface of the coordinates of component S (base) under the action the interface forces $\{F_I(t)\}$ as $\{\ddot{\delta}_{IS}(t)\}$. Thus, we can divide the base response analysis into 2 parts as shown in Fig. 3A

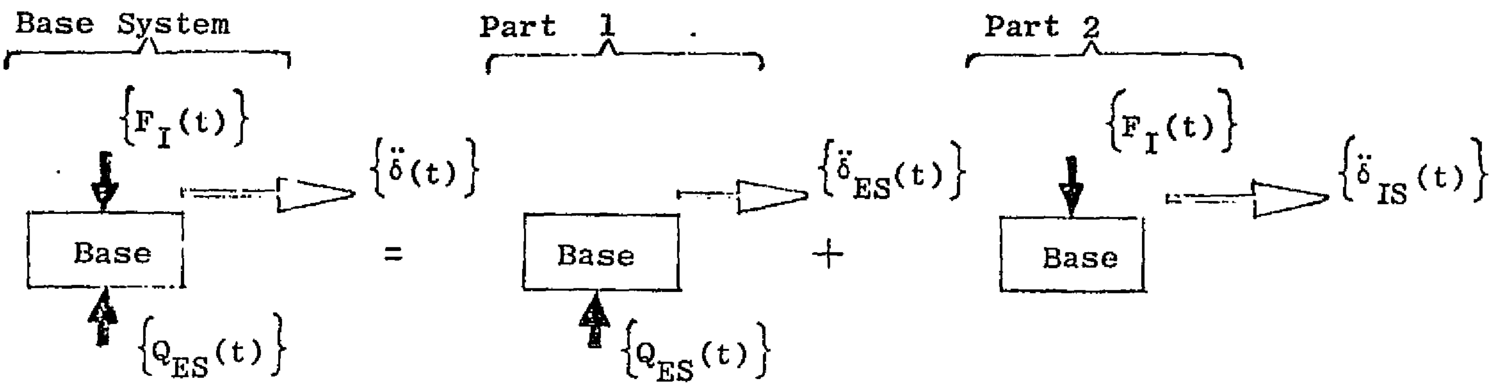


Fig. 3A

The application of superposition principle then yields

$$\{\ddot{\delta}(t)\} = \{\ddot{\delta}_{ES}(t)\} + \{\ddot{\delta}_{IS}(t)\} \tag{6}$$

At this stage, it is worthwhile mentioning that in defining the dynamic discrete model of the base system, interface masses of the branch will have to be lumped with the corresponding interface masses of the base. Thus the full inertia of the interface masses is accounted for. In the interest of separating branch analyses from support system analyses, it is suggested to use an interface branch point with no mass.

The response equations of the base system under the action of interface forces $\{F_I(t)\}$ (Part 2 of Fig.3A) is shown in Eq. (7).

$$\{\ddot{q}\}_S + \left[\,^\frown 2\rho\omega \,_\smile\right]_S \{\dot{q}\}_S + \left[\,^\frown \omega^2 \,_\smile\right]_S \{q\}_S = [\varphi]_S^T \{F_I(t)\} \qquad (7)$$

where $[\varphi]_S$ = mode shapes of component S normalized to yield unity generalized mass matrix.

Thus the interface acceleration due to $\{F_I(t)\}$ can be expressed as

$$\left\{\ddot{\delta}_{FI}(t)\right\} = \left[\varphi'\right]_S \left\{\ddot{q}\right\}_S \tag{8}$$

where $\left[\varphi'\right]_S$ = The pertinent rows of $\left[\varphi\right]_S$ matrix corresponding to the interface coordinates only.

Substituting Eq. (8) into Eq. (6) yields

$$\left\{\ddot{\delta}(t)\right\} = \left\{\ddot{\delta}_{ES}(t)\right\} + \left[\varphi'\right]_S \left\{\ddot{q}\right\}_S \tag{9}$$

The interface forces $\left\{F_I(t)\right\}$ can be computed from the inertia forces of the branch as follows:

$$\left\{F_I(t)\right\} = - \left[\Gamma\right] \left[M\right]_B \left\{\ddot{x}\right\}_B \tag{10}$$

where $\left[\Gamma\right]$ = matrix that relates inertially relieved forces to interface forces.

Substituting Eqs. (5) and (9) into (10) yields

$$\left\{F_I(t)\right\} = - \left[\Gamma\right] \left[M\right]_B \Big(\left[\beta\right] \left\{\ddot{\delta}_{ES}(t)\right\} + \left[\beta\right] \left[\varphi'\right]_S \left\{\ddot{q}\right\}_S + \left[\varphi_C\right]_B \left\{\ddot{q}\right\}_B\Big)$$

which can be rewritten as

$$\left\{F_I(t)\right\} = \left[TB\right] \left\{\ddot{q}\right\}_B + \left[TS\right] \left\{\ddot{q}\right\}_S + \left[TI\right] \left\{\ddot{\delta}_{ES}\right\} \tag{11}$$

where

$$\left[TB\right] = - \left[\Gamma\right] \left[M\right]_B \left[\varphi_C\right]_B$$

$$\left[TS\right] = - \left[\Gamma\right] \left[M\right]_B \left[\beta\right] \left[\varphi'\right]_S$$

$$\left[TI\right] = - \left[\Gamma\right] \left[M\right]_B \left[\beta\right]$$

Substituting Eq. (11) into Eq. (7), yields the response of the base in the coupled system:

$$\begin{aligned} \left\{\ddot{q}\right\}_S + \left[\diagdown 2\rho\omega \diagdown\right]_S \left\{\dot{q}\right\}_S + \left[\diagdown \omega^2 \diagdown\right]_S \left\{q\right\}_S = \left[\varphi'\right]_S^T \Big(\left[TB\right] \left\{\ddot{q}\right\}_B \\ + \left[TS\right] \left\{\ddot{q}\right\}_S + \left[TI\right] \left\{\ddot{\delta}_{ES}\right\} \Big) \end{aligned} \tag{12}$$

The response of the branch in the coupled system is obtained by substituting Eq. (9) into (4), yielding:

$$\begin{aligned} \left\{\ddot{q}\right\}_B + \left[\diagdown 2\rho\omega \diagdown\right]_B \left\{\dot{q}\right\}_B + \left[\diagdown \omega^2 \diagdown\right]_B \left\{q\right\}_B = \\ - \left[\varphi_C\right]_B^T \left[M\right]_B \left[\beta\right] \left\{\ddot{\delta}_{ES}\right\} \\ - \left[\varphi_C\right]_B^T \left[M\right]_B \left[\beta\right] \left[\varphi'\right]_S \left\{\ddot{q}\right\}_S \end{aligned} \tag{13}$$

Eqs. (12) and (13) represent a system of simultaneous - coupled equations which can be combined into one expression as follows:

$$\left[\begin{array}{c:c} \lceil I \rfloor & [\varphi_C]_B^T [M]_B [\beta] [\varphi']_S \\ \hdashline -[\varphi']_S^T [TB] & \lceil I \rfloor - [\varphi']_S^T [TS] \end{array}\right] \left\{\begin{array}{c} \ddot{q}_B \\ \hdashline \ddot{q}_S \end{array}\right\} + \left[\begin{array}{c:c} \lceil 2\rho\omega \rfloor_B & 0 \\ \hdashline 0 & \lceil 2\rho\omega \rfloor_S \end{array}\right] \left\{\begin{array}{c} \dot{q}_B \\ \hdashline \dot{q}_S \end{array}\right\}$$

$$+ \left[\begin{array}{c:c} \lceil \omega^2 \rfloor_B & 0 \\ \hdashline 0 & \lceil \omega^2 \rfloor_S \end{array}\right] \left\{\begin{array}{c} q_B \\ \hdashline q_S \end{array}\right\} = \left[\begin{array}{c} -[\varphi_C]_B^T [M]_B [\beta] \\ \hdashline [\varphi_S]^T [TI] \end{array}\right] \left\{\ddot{\delta}_{ES}(t)\right\} \tag{14}$$

Thus the transient response of the new system with an incompatible interface motion input $\left\{\delta_{ES}(t)\right\}$ requires the solution of Eq. (14)

With respect to part (a) of Fig. 1A the response of the original integrated base-branch system is known, i.e. we know $\left\{\ddot{\delta}(t)\right\}$ and $\left\{F_I(t)\right\}$. The response of the base system with this $\left\{F_I(t)\right\}$ acting can be expressed as

$$\left\{\ddot{q}'\right\}_S + \lceil 2\rho\omega \rfloor_S \left\{\dot{q}'\right\}_S + \lceil \omega^2 \rfloor_S \left\{q'\right\} = [\varphi']_S^T \left\{F_I(t)\right\} \tag{15}$$

Then it follows that

$$\left\{\ddot{\delta}_{IS}(t)\right\} = [\varphi']_S \left\{\ddot{q}'\right\}_S \tag{16}$$

The application of Eq. (6) now yields

$$\left\{\ddot{\delta}_{ES}(t)\right\} = \left\{\ddot{\delta}(t)\right\} - \left[\varphi'\right]_S \left\{\ddot{q}'\right\}_S \tag{17}$$

which removes the effect of the original branch from the integrated system.

APPENDIX B

Rigid Arm Mass Branch Coupling

The Rigid Arm Mass Branch Coupling

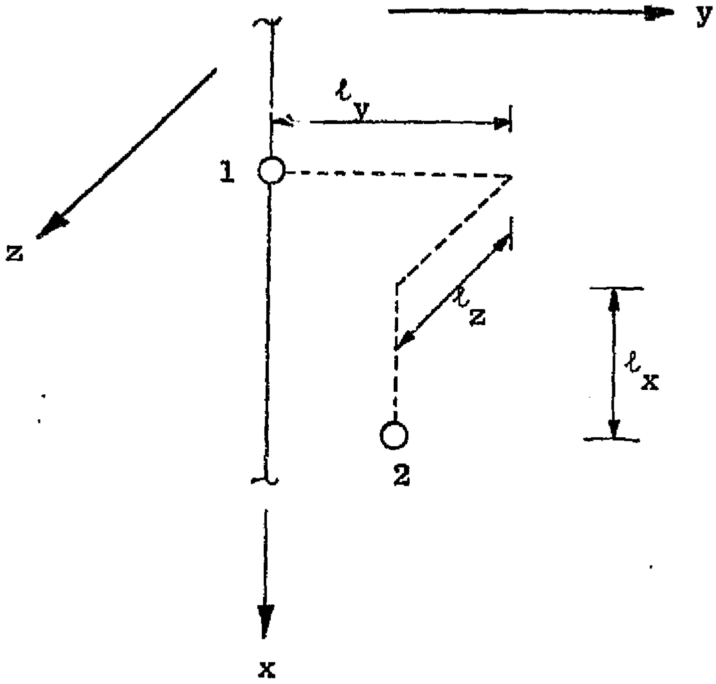


Fig. 1B

Considering an offset mass 2 as shown in Fig. 1B, the following definitions are assumed at 2

$$\{\delta_1\} = \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \end{Bmatrix}, \qquad \{\delta_2\} = \begin{Bmatrix} x_2 \\ y_2 \\ z_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{Bmatrix}$$

Right hand rule is taken to define θ_x, θ_y and θ_z

$$[M_2] = \left[\begin{array}{ccc|c} M_{x2} & & & \\ & M_{y2} & & 0 \\ & & M_{z2} & \\ \hline & 0 & & 0 \end{array}\right]$$

At 2, lumped mass in x, y and z direction are only taken.

The kinetic energy of the mass 2 can be written as

$$KE = \frac{1}{2} \{\dot{\delta}_2\}^T [M_2] \{\dot{\delta}_2\}. \tag{1}$$

$$KE = \frac{1}{2} \begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{z}_2 \\ \dot{\theta}_{x2} \\ \dot{\theta}_{y2} \\ \dot{\theta}_{z2} \end{Bmatrix}^T \left[\begin{array}{ccc|c} M_{x1} & & & \\ & M_{y1} & & 0 \\ & & M_{z1} & \\ \hline & 0 & & 0 \end{array}\right] \begin{Bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{z}_2 \\ \dot{\theta}_{x2} \\ \dot{\theta}_{y2} \\ \dot{\theta}_{z2} \end{Bmatrix} \tag{2}$$

But via the transformation

$$\begin{Bmatrix} x_2 \\ y_2 \\ z_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \ell_z & -\ell_y \\ 0 & 1 & 0 & -\ell_z & 0 & \ell_x \\ 0 & 0 & 1 & \ell_y & -\ell_x & 0 \\ \hline & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \end{Bmatrix} \tag{3}$$

or

$$\{\dot{\delta}_2\} = [TC]\{\dot{\delta}_1\} \tag{4}$$

$$[TC] = \left[\begin{array}{ccc|ccc} 1 & & & 0 & \ell_z & -\ell_y \\ & 1 & & -\ell_z & 0 & \ell_x \\ & & 1 & \ell_y & -\ell_x & 0 \\ \hline & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

Substituting Eq. (4) in Eq. (1) yields

$$= \frac{1}{2}\left([TC]\{\dot{\delta}_1\}\right)^T [M_2]\left([TC]\{\dot{\delta}_1\}\right)$$

$$= \frac{1}{2}\ \{\dot{\delta}_1\}^T [TC]^T [M_2][TC]\{\dot{\delta}_1\}$$

$$= \frac{1}{2}\ \{\dot{\delta}_1\}^T [MBC]\{\dot{\delta}_1\}$$

where $[MBC] = [TC]^T [M_2][TC]$ (5)

is the rigid arm mass branch coupling matrix, which describes the KE of mass 2 in terms of the velocity vectors of the degrees of freedom at point 1.

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